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GEOMETRICALLY NONLINEAR ANALYSIS OF LAYERED COMPOSITE PLATES AND SHELLS

by

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ABSTRACT

A degenerated three-dimensional finite element based on the total Lagrangian, incremental, formulation of a three-dimensional layered anisotropic medium is developed, and its use in the geometrically nonlinear, static as well as dynamic, analysis of layered composite plates and sheils is demonstrated via several example problems. For comparison purposes a two-dimensional finite element based on the Sanders shell theory with the von Karman (nonlinear) strains is also presented. The elements have the following features:

- Geometrically linear and nonlinear analysis
- Static and transient analyses
- Natural vibration (linear) analyses
- Plates and shell elements
- Arbitrary loading and boundary conditions
- Arbitrary lamination scheme and lamina properties

The element can be used, with minor changes, in any existing general purpose programs.

The 3-D dimensional degenerated element has computational simplicity over a fully three-dimensional element, and the element accounts for full geometric nonlienarities in contrast to the 2-dimensional elements based on the Sanders shell theory. As demonstrated

via numerical examples, the deflections obtained by the 2-D shell element deviate from those obtained by the 3-D element for deep shells. Further, the 3-D element can be used to model general shells that are not necessarily doubly-curved. For example, the twisted plates cannot be modeled using the 2-D shell element. Of course, the 3-D degenerated element is computationally more demanding than the 2-D shell theory element for a given problem. In summary, the present 3-D element is an efficient element for the analysis of layered composite plates and shells undergoing large displacements and transient motion.

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INTRODUCTION

1.1 Motivation

Composite materials and reinforced plastics are increasingly used in aircraft, space vehicles, automobiles, and pressure vessels. With the increased use of fiber-reinforced composites as structural elements, studies involving the thermomechanical behavior of shell components made of composites are receiving considerable attention. Functional requirements and economic considerations of design have forced designers to use accurate but economical methods of determining stresses, natural frequencies, buckling loads, etc.

An accurate prediction of the behavior of shell structures requires a realistic modeling of the actual geometry, material behavior, and kinematic description of the components. The partial differential equations describing the large-deflection behavior of anisotropic composite shells of arbitrary geometry are not amenable to classical analytical methods. Consequently, numerical and approximate methods must be used to predict desired design quantities (such as stresses, frequencies, and buckling loads). In the last two and a half decades the finite element method has emerged as the most powerful analysis method of structural analysis.

The majority of the research papers in the open literature on shells is concerned with bending, vibration, and buckling of isotropic shells. As composite materials are making their way into many engineering structures, analyses of shells made of such materials become important. Further, with the increased application of advanced fiber composites in jet engine fan or compressor blades and high performance

aircraft, studies involving transient response of composite shell structures are needed to assess the capability of these materials under dynamic loads. At the time the project was undertaken, a finite-element analysis of the nonlinear transient response of laminated anisotropic shells was not available.

Finite-element analyses of shell structures in the past have used one of the three types of elements: 1. A 2-dimensional (2-D) element based on a two-dimensional shell theory; 2. A 3-D element based on three-dimensional elasticity theory; and 3. A 3-D degenerated element derived from the 3-D elasticity theory. The 2-D shell theory is derived form the three dimensional continuum field equations via, for example, an analytical integration through the thickness is employed to reduce the theory to a two-dimensional theory. In doing so the static and kinematic resultants are defined and used to derive the equations. In contrast to the 2-D shell theory, in the 3-D degenerated element, the shell geometry and displacement fields are discretized from the outset in the sense of finite elements, and the element contains full geometric nonlinearity. The unavailability of a convenient, general nonlinear shell theory makes the 2-D shell element restrictive in its use. The nonlinearity included in the 2-D shell element is that due to the von Karman strains, in which the products of the derivatives of the transverse deflection are neglected. In contrast to the 2-D shell theory, no specific shell theory is employed in the 3-D degenerated element; instead, the geometry and the displacement fields are directly discretized and interpolated as in the analysis of continuum problems. The 2-D elements based on shell theory are the most economical, followed by the 3-D degenerated element. While the 3-D element is most accurate,

at least in theory, the computational cost prohibits its use in the nonlinear transient analysis of shells.

The present study is motivated by the lack of a finite-element analysis of geometrically nonlinear transient response of laminated anisotropic shells. The present study involved the development of a 3-D degenerated shell element for the analysis of a layered anisotropic medium, accounting for full geometric nonlinearity and transverse normal and shear stresses. The following literature survey provides a background for the present work.

1.2 A Review of the Literature

There exist a number of theories for layered anisotropic shells. Many of these theories were developed originally for thin shells, and are based on the Kirchhoff-Love kinematic hypothesis that plane sections normal to the undeformed midsurface remain plane and normal after deformation. Excellent surveys of various shell theories can be found in the works of Naghdi [1] and Bert [2]. Here we review the literature on composite shells.

The first analysis that incorporated the bending-stretching coupling (due to unsymmetric lamination in composites) is due to Ambartsumyan [3,4]. In his analyses Ambartsumyan assumed that the individual orthotropic layers were oriented such that the principal axes of material symmetry coincided with the principal coordinates of the shell reference surface. Thus, Ambartsumyan's work dealt with what is now known as laminated orthotropic shells rather than laminated anisotropic shells; in laminated anisotropic shells the individual layers are, in general, anisotropic and the principal axes of material

symmetry of the individual layers do not necessarily coincide with the principal coordinates of the shell.

In 1962 Dong, Pister and Taylor [5] formulated a theory of thin shells laminated of anisotropic material. The theory is an extension of the theory developed by Stavsky [6] for laminated anisotropic plates to Donnell's shallow shell theory. Cheng and Ho [7] presented an analysis of laminated anisotropic cylindrical shells using Flügge's shell theory. A first approximation theory for the unsymmetric deformation of nonhomogeneous, anisotropic, elastic cylindrical shells was derived by Widera and Chung [8] by means of the asymptotic integration of the elasticity equations. For a homogeneous, isotropic material, the theory reduces to the Donnell's equations.

All of the theories discussed above are based on the Kirchhoff-Love hypotheses in which the transverse shear deformation is neglected. The Love's first approximation theories are expected to yield sufficiently accurate results when (i) the lateral dimension to thickness ratio is large; (ii) the dynamic excitations are within the low-frequency range; (iii) the material anisotropy is not severe. However, application of such theories to layered anisotropic composite shells could lead to as much as 30% or more errors in deflections, stresses, or frequencies. For example, the thick-walled composite cylinders used for aircraft landing gears require a laminated thick-shell theory for their analysis.

The effect of transverse shear deformation and transverse isotropy, as well as thermal expansion through the shell thickness were considered by Gulati and Essenberg [9] and Zukas and Vinson [10]. Gulati and Essenberg [9] showed that the circumferential displacement components and the twist couple would arise due to the anisotropy and transverse

shear deformation. Whitney and Sun [11] developed a shear deformable theory for laminated cylindrical shells that includes both transverse shear deformation and transverse normal strain as well as expansional strains. Recently, Widera and Logan [12,13] presented refined theories for nonhomogeneous anisotropic cylindrical shells.

As far as the finite-element analysis of shells is concerned, layered composite shells have not received nearly as much attention as ordinary shells. The works of Dong [14] on statically-loaded orthotropic shell of revolution, Dong and Selna [15] on free vibration of the same, Wilson and Parsons [16] on static axisymmetric loading of arbitrarily thick orthotropic shells of revolution, and Schmit and Monforton [17] on laminated anisotropic cylindrical shells are the only ones that considered the finite element method before the 1970's (note that the latter reference is the only one that considered laminated anisotropic shells). In the 1970's there was an increased interest in the finite-element analysis of bending and vibration of laminated anisotropic shells. Apparently the first finite-element application in laminated anisotropic shells of arbitrary geometry is due to Thompson and Bert [18], who treated free (i.e., natural) vibration of general laminated anisotropic thin shells. Other finite-element analyses of layered anisotropic composite shells include the works of Panda and Natarajan [19], Shivakumar and Krishna Murty [20], Rao[21], Seide and Chang [22], and Venkatesh and Rao [23]. Recently, Hsu, et al. [24] and Reddy [25] presented finite-element analyses of laminated thick cylindrical shells and laminated thick doubly curved shells, respectively.

All of the literature cited above is limited to the small displacement theory of shells. In the analysis of thin flexible composite shells one should take large deflections into account. Because of the high modulus and high strength properties that composites have, structural components could undergo large deflections before they become inelastic. Therefore, an accurate prediction of displacements and stresses is possible only when one accounts for the geometric nonlinearity.

Finite-element analyses of the large-displacement theory are based on the principle of virtual work or the associated principle of stationary potential energy. Horrigmoe and Bergan [26] presented classical variational principles for nonlinear problems by considering incremental deformations of a continuum. A survey of various principles in incremental form in different reference configurations, such as the total Lagrangian and the updated Lagrangian formulation, is presented by Wunderlich[27]. In the total Lagrangian description, all static and kinematic variables are referred to the initial configuration. In the updated Lagrangian description all variables are referred to the current configuration. Stricklin et al. [28] presented a survey of various formulations and solution procedures for nonlinear static and dynamic structural analysis. The formulations included are the pseudo-force method, total Lagrangian method, the updated Lagrangian method, and the convected coordinate method. The solution methods included are the solution by direct minimization of total potential, Newton-Raphson and modified Newton-Raphson, and the first- and second-order self correcting methods.

The only large-deflection analyses of laminated composite shells that can be found in the literature are the static analysis of Noor and Hartley [29] and Chang and Sawamiphakdi [30]. Noor and Hartley employed the shallow shell theory with transverse shear strains and geometric nonlinearities to develop triangular and quadrilateral finite elements. Chang and Sawamiphakdi presented a formulation of the 3-D degenerated element for geometrically nonlinear analysis of laminated composite shells. The formulation is based on the updated Lagrangian description and it does not include any numerical results for laminated shells.

From the review of the literature it is clear that the 3-D degenerated element has not been exploited fully for geometrically nonlinear analysis of laminated anisotropic shells. Further, the transient analysis has not been reported in the literature. Guided by these observations the present work was undertaken in the fall of 1981.

1.3 The Present Study

The present study was undertaken to develop a finite-element analysis capability for the static and dynamic analysis of geometrically nonlinear theory of layered anisotropic shells. The 3-D degenerated element with total Lagrangian description is used to analyze various shell problems.

Following this introduction, a description of the 2-D shell element is presented in Chapter 2. In Chapter 3 a detailed discussion of the 3-D degenerated element is given. Application and comparison of the two elements are illustrated via a number of shell problems.

Chapter II

A SHEAR DEFORMABLE SHELL ELEMENT

2.1 Governing Equations

The basic equations of three-dimensional elasticity theory can be simplified for thin flexible bodies. A set of simplifying assumptions that provide a reasonable description of the behavior of thin elastic shells are as follows:

1. the thickness of the shell is small compared to the other dimensions;
2. the transverse normal stress is negligible;
3. normals to the reference surface of the shell before deformation remain straight but not necessarily normal after deformation;
4. the thickness-to-radius of the shell is assumed to be small compared to unity; and
5. in the second order terms, the derivatives of membrane displacements are small compared to the derivatives of the transverse displacement.

The shell under consideration is composed of a finite number of orthotropic layers of uniform thickness, as shown in Fig. 2.1. An orthogonal curvilinear coordinate system (ξ_1,ξ_2,ζ) is chosen such that the ξ_1- and ξ_2- curves are lines of principal curvature on the midsurface $\zeta = 0$, and ζ-curves are straight lines perpendicular to the surface $\zeta = 0$. A line element of the shell is given by (see [31])

$$(ds)^2 = [(1 + \zeta/R_1)\alpha_1 d\xi_1]^2 + [(1 + \zeta/R_2)\alpha_2 d\xi_2]^2 + (d\zeta)^2 \quad (2.1)$$

where α_i and R_i $(i = 1,2)$ are the surface metrics and radii of

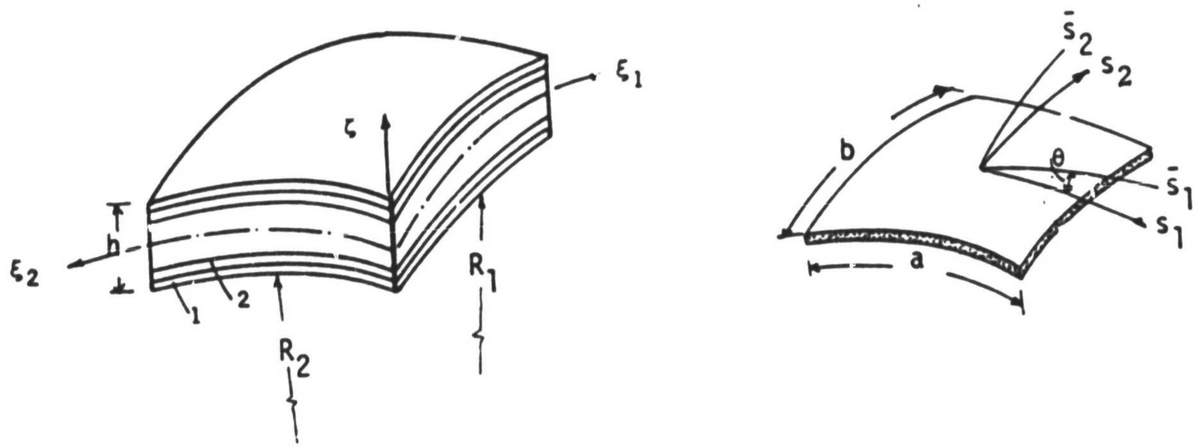

Figure 2.1 Laminated shell geometry and lamina details

curvature, respectively. In general, α_i and R_i are functions of ξ_i only.

For constant values of α_1, α_2, R_1, and R_2, the equations of motion are given by (with $c_o = \frac{1}{2}(1/R_1 - 1/R_2)$ and $dx_i = \alpha_i d\xi_i$; see [25])

$$\frac{\partial N_1}{\partial x_1} + \frac{\partial}{\partial x_2}(N_6 + c_o M_6) + \frac{Q_1}{R_1} = P_1 \frac{\partial^2 u_1}{\partial t^2} + [P_2 + P_3 (\frac{1}{R_1} + \frac{1}{R_2})] \frac{\partial^2 \phi_1}{\partial t^2}$$

$$\frac{\partial}{\partial x_1}(N_6 - c_o M_6) + \frac{\partial N_2}{\partial x_2} + \frac{Q_2}{R_2} = P_1 \frac{\partial^2 u_2}{\partial t^2} + [P_2 + P_3 (\frac{1}{R_1} + \frac{1}{R_2})] \frac{\partial^2 \phi_2}{\partial t^2}$$

$$\frac{\partial Q_1}{\partial x_1} + \frac{\partial Q_2}{\partial x_2} - (\frac{N_1}{R_1} + \frac{N_2}{R_2} - q) = [P_1 + P_2 (\frac{1}{R_1} + \frac{1}{R_2})] \frac{\partial^2 u_3}{\partial t^2}$$

$$\frac{\partial M_1}{\partial x_1} + \frac{\partial M_6}{\partial x_2} - Q_1 = P_3 \frac{\partial^2 \phi_1}{\partial t^2} + [P_2 + P_3 (\frac{1}{R_1} + \frac{1}{R_2})] \frac{\partial^2 u_1}{\partial t^2}$$

$$\frac{\partial M_6}{\partial x_1} + \frac{\partial M_2}{\partial x_2} - Q_2 = P_3 \frac{\partial^2 \phi_2}{\partial t^2} + [P_2 + P_3 (\frac{1}{R_1} + \frac{1}{R_2})] \frac{\partial^2 u_2}{\partial t^2} \qquad (2.2)$$

where u_i are the displacements of the reference surface along the ξ_i and ζ axes, ϕ_i are the rotations of the reference surface with respect to the ξ_i -axis, q is the distributed load, N_i and M_i are the stress and moment resultants, and Q_i is the shear force resultant:

$$(N_i, M_i) = \sum_{k=1}^{L} \int_{\zeta_{k-1}}^{\zeta_k} \sigma_i (1, \zeta) d\zeta \quad , \quad i = 1,2,6$$

$$Q_i = \sum_{k=1}^{L} K_i^2 \int_{\zeta_{k-1}}^{\zeta_k} \sigma_i d\zeta \quad , \quad i = 4,5, \qquad (2.3)$$

Here P_i are the inertias

$$(P_1, P_2, P_3) = \sum_{k=1}^{L} \int_{\zeta_{k-1}}^{\zeta_k} \rho^{(k)} (1, \zeta, \zeta^2) d\zeta \tag{2.4}$$

and K_i ($i = 4,5$) are shear correction factors.

The strain-displacement relations for a large rotation (small strains) theory of shells are given by (see [32])

$$\begin{aligned}
\varepsilon_1 &= \varepsilon_1^0 + \zeta\kappa_1 \\
\varepsilon_2 &= \varepsilon_2^0 + \zeta\kappa_2 \\
\varepsilon_4 &= \varepsilon_4^0 \\
\varepsilon_5 &= \varepsilon_5^0 \\
\varepsilon_6 &= \varepsilon_6^0 + \zeta\kappa_6
\end{aligned} \tag{2.5}$$

where

$$\varepsilon_1^0 = \frac{\partial u_1}{\partial x_1} + \frac{u_3}{R_1} + \frac{1}{2} [(\frac{\partial u_3}{\partial x_1} + \underline{\frac{u_1}{R_1}})^2 + \underline{(\frac{\partial u_1}{\partial x_1} + \frac{u_3}{R_1})^2} + \underline{(\frac{\partial u_2}{\partial x_1})^2}]$$

$$\varepsilon_2^0 = \frac{\partial u_2}{\partial x_2} + \frac{u_3}{R_2} + \frac{1}{2} [(\frac{\partial u_3}{\partial x_2} + \underline{\frac{u_2}{R_2}})^2 + \underline{(\frac{\partial u_2}{\partial x_2} + \frac{u_3}{R_2})^2} + \underline{(\frac{\partial u_1}{\partial x_2})^2}]$$

$$\varepsilon_6^0 = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} + (\frac{\partial u_3}{\partial x_1} + \underline{\frac{u_1}{R_1}})(\frac{\partial u_3}{\partial x_2} + \underline{\frac{u_2}{R_2}}) + \underline{(\frac{\partial u_1}{\partial x_2} + \frac{u_3}{R_1}) \frac{\partial u_1}{\partial x_2}} + \underline{(\frac{\partial u_2}{\partial x_1} + \frac{u_3}{R_2}) \frac{\partial u_2}{\partial x_1}}$$

$$\varepsilon_4^0 = \phi_2 + \frac{\partial u_3}{\partial x_2} - \frac{u_2}{R_2}$$

$$\varepsilon_5^0 = \phi_1 + \frac{\partial u_3}{\partial x_1} - \frac{u_1}{R_1}$$

$$\kappa_1 = \frac{\partial \phi_1}{\partial x_1} + \frac{1}{R_1} (\frac{\partial u_1}{\partial x_1} + \frac{u_3}{R_1})$$

$$\kappa_2 = \frac{\partial \phi_2}{\partial x_2} + \frac{1}{R_2}\left(\frac{\partial u_2}{\partial x_2} + \frac{u_3}{R_2}\right)$$

$$\kappa_6 = \frac{\partial \phi_1}{\partial x_1} + \frac{\partial \phi_2}{\partial x_2} - c_o\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) \qquad (2.6)$$

Invoking the fourth assumption, the contribution from the underlined terms in Eq. (2.6) can be neglected. The transverse normal strain and normal stresses are neglected in the present theory.

The shell constitutive equations are given by

$$\begin{aligned} N_i &= A_{ij}\varepsilon_j^o + B_{ij}\kappa_j \\ M_i &= B_{ij}\varepsilon_j^o + D_{ij}\kappa_j \end{aligned} \qquad (i,j = 1,2,6)$$

$$Q_2 = A_{44}\varepsilon_4^o + A_{45}\varepsilon_5^o$$

$$Q_1 = A_{45}\varepsilon_4^o + A_{55}\varepsilon_5^o \qquad (2.7)$$

Here A_{ij}, B_{ij} and D_{ij} $(i,j = 1,2,6)$ denote the extensional,* flexural-extensional coupling, and flexural stiffnesses:

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{L} \int_{\zeta_{k-1}}^{\zeta_k} Q_{ij}^{(k)} (1, \zeta, \zeta^2) d\zeta . \qquad (2.8)$$

where L denotes the total number of layers, and $Q_{ij}^{(k)}$ are the plane-stress reduced stiffnesses of the k-th lamina, referred to the shell axes.

Equations (2.1)-(2.8) completely describe the dynamic equilibrium of a layered, anisotropic shell. These equations can be solved in closed form for the small displacement theory of simple-supported,

*Quantities A_{44}, A_{45}, A_{55} and A_{66} are actually shear stiffnessess, not extensional ones.

cross-ply plates under sinusoidal distribution of the transverse load (see [33,34]). In order to solve practically important problems that involve other loadings, boundary conditions, geometry, and lamination schemes, one must consider approximate method of analysis. In the following section, an isoparameteric finite-element analysis of the shell equations (2.1)-(2.8) is presented.

2.2 Finite-Element Model

A typical finite element is a doubly-curved shell element whose projections on the ξ_1-ξ_2-plane is an isoparametric quadrilateral element. Over the typical shell element $\Omega^{(e)}$, the displacements ($u_1, u_2, u_3, \phi_1, \phi_2$) are interpolated by expressions of the form,

$$u_i = \sum_{j=1}^{N} u_i^j \psi_j(\xi_1,\xi_2) \quad , \quad i = 1,2,3$$

$$\phi_i = \sum_{j=1}^{N} \phi_i^j \psi_j(\xi_1,\xi_2) \quad , \quad i = 1,2 \tag{2.9}$$

where ψ_j are the interpolation functions, and u_i^j and ϕ_i^j are the nodal values of u_i and ϕ_i, respectively. For a linear isoparametric element with nine nodes (N=9), this interpolation results in a stiffness matrix of order 45 by 45.

Substitution of Eq. (2.9) into the variational formulation of Eq. (2.2) yields an element equation of the form (see Reddy [34])

$$[K]\{\Delta\} + [M]\{\ddot{\Delta}\} = \{F\} \tag{2.10}$$

where $\{\Delta\} = \{\{u_1\}, \{u_2\}, \{u_3\}, \{\phi_1\}, \{\phi_2\}\}^T$, [K] and [M] are element stiffness and mass matrices, respectively, and {F} is the force vector. In the interest of brevity, the coefficients of mass and stiffness matrices are included in Appendix I.

To complete the approximation, we should approximate the time derivatives in Eq. (2.10). Here we use the Newmark direct integration scheme (the constant-average-acceleration method). Use of the Newmark method to Eq.(2.10) yields (see Reddy [34])

$$[\hat{K}]\{\Delta\}_{n+1} = \{\hat{F}\}_{n,n+1} \tag{2.11}$$

where

$$[\hat{K}] = [K] + a_0[M],\ \{\hat{F}\} = \{F\}_{n+1} + [M](a_0\{\Delta\}_n + a_1\{\dot{\Delta}\}_n + a_2\{\ddot{\Delta}\}_n),$$

$$a_0 = 1/(\beta\Delta t^2) \quad , \quad a_1 = a_0\Delta t \quad , \quad a_2 = \frac{1}{2\beta} - 1. \tag{2.12}$$

Once the solution $\{\Delta\}$ is known at $t_{n+1} = (n+1)\Delta t$, the first and second derivatives (velocity and accelerations) of $\{\Delta\}$ at t_{n+1} can be computed from

$$\{\ddot{\Delta}\}_{n+1} = a_0(\{\Delta\}_{n+1} - \{\Delta\}_n) - a_1\{\dot{\Delta}\}_n - a_2\{\ddot{\Delta}\}_n$$
$$\{\dot{\Delta}\}_{n+1} = \{\dot{\Delta}\}_n + a_3\{\ddot{\Delta}\}_n + a_4\{\ddot{\Delta}\}_{n+1} \tag{2.13}$$

where $a_3 = (1 - \alpha)\Delta t$, and $a_4 = \alpha\Delta t$.

The element equations (2.11) can be assembled, boundary conditions can be imposed, and the resulting equations can be solved at each time step using the information known from the preceding time step solution. At time $t = 0$, the initial values of $\{\Delta\}$, $\{\dot{\Delta}\}$, and $\{\ddot{\Delta}\}$ (obtained by solving Eq. (2.10) at $t = 0$) are used to initiate the time marching scheme.

In the present study the nine-node rectangular isoparametric element was employed. Analogous to the shear deformable theory of layered composite plates [34], the present theory can be recognized as a shear deformable theory derived from the classical shell theory by treating the slope-deflection relations ($\varepsilon_4, \varepsilon_5 = 0$) as constraints, and

including the constraints into the variational formulation of the shell equations by the penalty function method (see Reddy [25]). The elements derived using such theory are very stiff (so-called locking is observed) for thin shells, but yield good results for moderately thick shells. To overcome the locking phenomenon, the reduced integration technique (see Zienkiewicz, Taylor and Too [35]) must be employed in the evaluation of the stiffness coefficients associated with the shear energy terms (i.e., penalty terms). More specifically, the 2x2 Gauss rule must be used for shear terms (i.e., those involving A_{44}, A_{45}, and A_{55}) and the standard 3x3 Gauss rule must be used for the bending terms when the nine node quadratic isoparametric element.

Chapter III

DEGENERATED THREE DIMENSIONAL FINITE ELEMENT

3.1 Introduction

The primary objective of this chapter is to review the formulation of equations governing geometrically nonlinear motion of a continuous medium. Due to the nature of the present manuscript, only necessary equations are presented. For additional details the reader is referred to [36-40].

We describe the motion of a continuous body in a Cartesian coordinate system. The simultaneous position of all material points (i.e., the configuration) of the body at time t is denoted by C_t, and C_0 and $C_{t+\Delta t}$ denote the configurations at reference time $t = t_0$ and time $t + \Delta t$, respectively (see Fig. 3.1). In the updated Lagrangian description all kinetic and kinematic variables are referred to the current configuration at each time and load step. In the total Lagrangian description all dependent variables are referred to the reference configuration. The updated Lagrangian is more suitable for motions that involve very large distortions of the body (e.g., high-velocity impact). The total Lagrangian is more convenient for motions that involve only moderately large deformations. In the present study the total Lagrangian formulation is adopted.

3.2 Formulation of the Incremental Equations of Motion

Here we present a derivation of the equilibrium equations at different time steps using the total Lagrangian approach. The coordinates of a typical point in C_t is denoted by ${}^t\underset{\sim}{x} = ({}^tx_1, {}^tx_2, {}^tx_3)$. The displacement of a particle at time t is given by

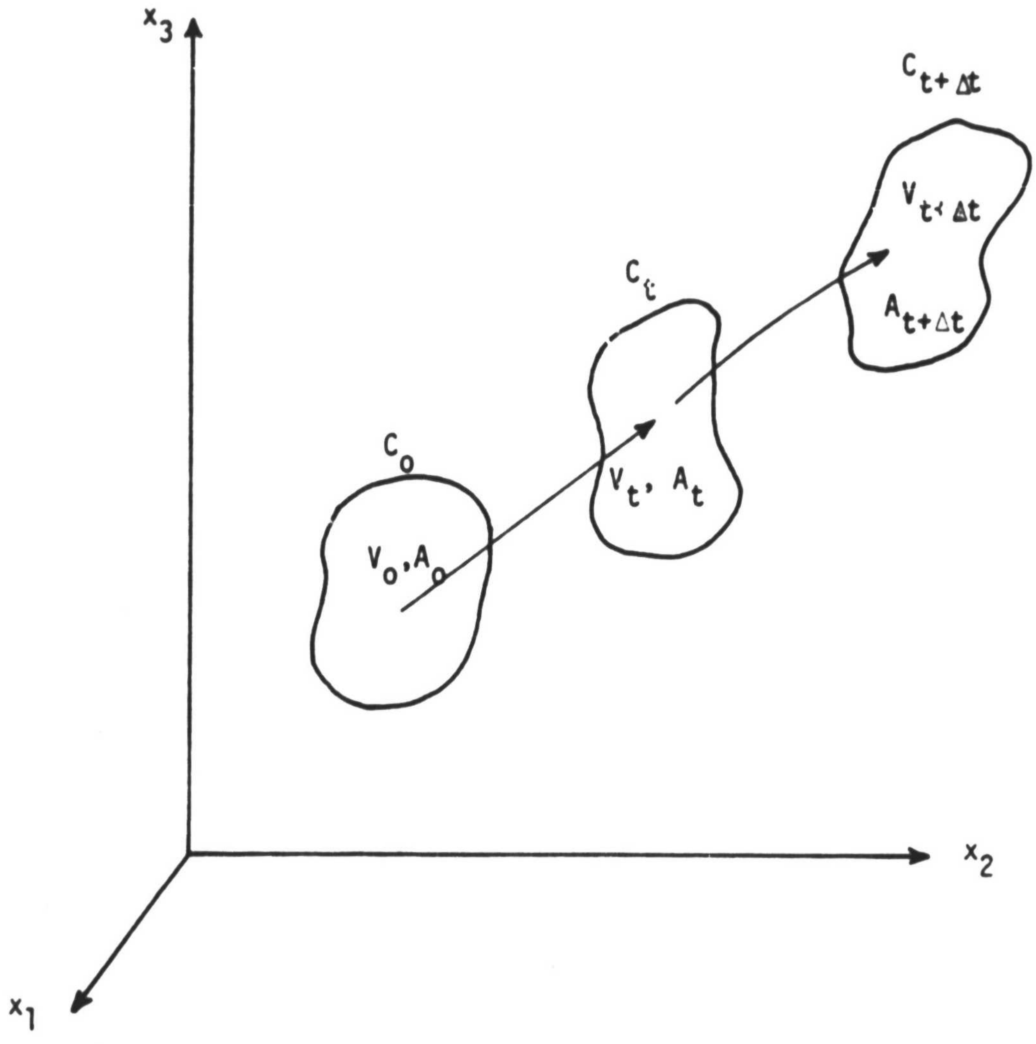

Figure 3.1 Motion of a continuous body in Cartesian coordinates

$$ {}^{t}\underset{\sim}{u} = {}^{t}\underset{\sim}{x} - {}^{0}\underset{\sim}{x} \text{ or } {}^{t}u_i = {}^{t}x_i - {}^{0}x_i \tag{3.1}$$

The increment of displacement during time t to t + Δt is defined by

$$u_i = {}^{t+\Delta t}u_i - {}^{t}u_i \tag{3.2}$$

The principle of virtual displacements can be employed to write the equilibrium equations at any fixed time t. The principle, applied to the large-displacements case, can be expressed mathematically as follows:

$$\int_{V_0} \rho_0 \, {}^{t+\Delta t}\ddot{u}_i \, \delta u_i \, dV_0 + \int_{V_0} {}^{t+\Delta t}S_{ij} \, \delta({}^{t+\Delta t}\varepsilon_{ij}) dV_0$$

$$= \int_{A_0} {}^{t+\Delta t}T_i \, \delta u_i \, dA_0 + \int_{V_0} {}^{t+\Delta t}F_i \, \delta u_i \, dV_0 \tag{3.3}$$

where summation on repeated indices is implied; V_0, A_0, and ρ_0 denote, respectively, a volume element, area element, and density in the initial configuration, S_{ij} are the components of the second Piola-Kirchhoff stress tensor, ε_{ij} the components of the Green-Lagrangian strain tensor, T_i the components of boundary stresses, and F_i are the components of the body force vector. The superposed dots on u_i denote differentiation with respect to time, and δ denotes the variational symbol. In writing Eq. (3.3) it is assumed that ε_{ij} is related to the displacement components by the kinematic relations

$${}^{t+\Delta t}\varepsilon_{ij} = \frac{1}{2}\left({}^{t+\Delta t}u_{i,j} + {}^{t+\Delta t}u_{j,i} + {}^{t+\Delta t}u_{m,i} \, {}^{t+\Delta t}u_{m,j}\right) \tag{3.4}$$

where $u_{i,j} = \partial u_i/\partial x_j$. The strain components ${}^{t+\Delta t}\varepsilon_{ij}$ can be expressed in terms of current strain and incremental strain components:

$$^{t+\Delta t}\epsilon_{ij} = \frac{1}{2}\left({}^{t}u_{i,j} + {}^{t}u_{j,i} + {}^{t}u_{m,i}\,{}^{t}u_{m,j}\right)$$

$$+ \frac{1}{2}\left(u_{i,j} + u_{j,i} + {}^{t}u_{m,i}\,u_{m,j} + u_{m,i}\,{}^{t}u_{m,j}\right) + \frac{1}{2}u_{m,i}\,u_{m,j}$$

$$\equiv {}^{t}\epsilon_{ij} + (e_{ij} + \eta_{ij}) \tag{3.5}$$

where e_{ij} and η_{ij} denote the linear and nonlinear incremental strains. The stress components ${}^{t+\Delta t}S_{ij}$ can be decomposed into two parts:

$$^{t+\Delta t}S_{ij} = {}^{t}S_{ij} + S_{ij} \tag{3.6}$$

where S_{ij} is the incremental stress tensor. The incremental stress components S_{ij} are related to the incremental Green-Lagrange strain components, $\epsilon_{ij} = e_{ij} + \eta_{ij}$, by the generalized Hooke's law:

$$S_{ij} = C_{ijk\ell}\epsilon_{k\ell}, \tag{3.7}$$

where $C_{ijk\ell}$ are the components of the elasticity tensor. Using Eq. (3.4)-(3.7), one can be express Eq. (3.3) in the alternate form

$$\int_{V_o} \rho_o\, {}^{t+\Delta t}\ddot{u}_i\,\delta u_i\, dV_o + \int_{V_o} C_{ijk\ell}(e_{k\ell}\delta\eta_{ij} + \eta_{k\ell}\delta e_{ij})dV_o$$

$$+ \int_{V_o} {}^{t}S_{ij}\,\delta e_{ij}\, dV_o = \delta W - \int_{V_o} {}^{t}S_{ij}\,\delta\eta_{ij}\, dV_o \tag{3.8}$$

where δW is the virtual work due to external loads.

3.3 Finite-Element Formulation

3.3.1 Geometry of the Element

Consider a solid three-dimensional element shown in Fig. 3.2. The coordinates of a typical point in the element can be written as

$$x_i = \sum_{j=1}^{n} \phi_j(\xi_1,\xi_2)\,\frac{1+\zeta}{2}\,(x_i^j)_{top} + \sum_{j=1}^{n} \phi_j(\xi_1,\xi_2)\,\frac{1-\zeta}{2}\,(x_i^j)_{bottom} \tag{3.9}$$

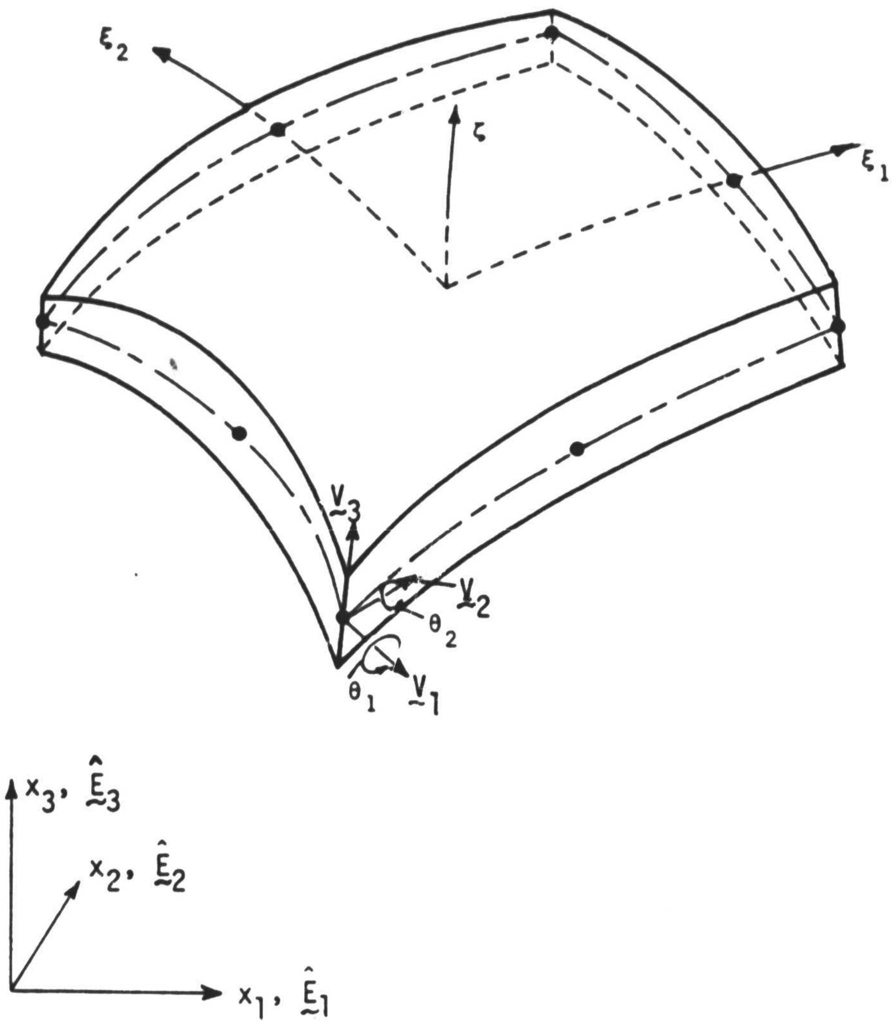

Figure 3.2 Geometry of the degenerated three-dimensional element

where n is the number of nodes, $\phi_i(\xi_1,\xi_2)$ are the finite-element interpolation (or shape) functions, which, in the element take the value of unity at node i and zero at all other nodes, ξ_1 and ξ_2 are the normalized curvilinear coordinates in the middle plane of the shell, and ζ is a linear coordinate in the thickness direction and x_1^i, x_2^i, and x_3^i are the global coordinates at node i. Here ξ_1, ξ_2, and ζ are assumed to vary between -1 and +1. Now let (see Fig. 3.2)

$$v_{3k}^i = (x_k^i)_{top} - (x_k^i)_{bottom} \tag{3.10}$$

$$\hat{\underset{\sim}{e}}_3^i \equiv \underset{\sim}{v}_3^i / |\underset{\sim}{v}_3^i|$$

where v_{3k}^i is the k-th component of the vector $\underset{\sim}{v}_3^i$. Then Eq. (3.9) becomes

$$x_i = \sum_{j=1}^{n} [\phi_j (x_i^j)_{mid} + \phi_j \frac{\zeta}{2} h_j \hat{e}_{3i}^j] \tag{3.11}$$

where h_j is the thickness of the element at node j. For small deformation, the displacement of every point in the element can be written as

$$u_i = \sum_{j=1}^{n} \phi_j [u_i^j + \zeta \frac{h}{2} (\hat{e}_{1i}^j \theta_2^j - \hat{e}_{2i}^j \theta_1^j)] \tag{3.12}$$

where θ_1^i and θ_2^i are the rotations about (local) unit vectors $\hat{\underset{\sim}{e}}_1^i$ and $\hat{\underset{\sim}{e}}_2^i$, respectively, u_1, u_2, and u_3 are the displacement components corresponding to the global coordinate x_1, x_2, x_3 directions respectively, and u_1^i, u_2^i and u_3^i are the values of the displacements

(referred to $\underset{\sim}{x}$) at node i. In writing Eq. (3.12), we assumed that a line that is straight and normal to the middle surface before deformation is still straight but not necessarily 'normal' to the middle surface after deformation. The strain energy corresponding to stress perpendicular to the middle surface is ignored to improve numerical conditioning when the three-dimensional element is employed. This constraint corresponds only to a part of the usual assumptions of a two-dimensional shell theory. The relaxation of the requirement that straight lines perpendicular to the middle surface remain normal to the deformed middle surface permits the shell to experience shear deformation - an important feature in thick shell situations.

3.3.2 Displacement Field in the Element

In the present study the current coordinates ${}^t x_i$ are interpolated by the expression

$$ {}^t x_i = \sum_{j=1}^{n} \phi_j ({}^t x_i^j + \frac{1}{2} \zeta h_j \; {}^t \hat{e}_{3i}^j) \tag{3.13} $$

and the displacement by

$$ {}^t u_i = \sum_{j=1}^{n} \phi_j \; [{}^t u_i^j + \frac{1}{2} \zeta h_j \; ({}^t \hat{e}_{3i}^j - {}^o e_{3i}^j)] \tag{3.14} $$

$$ u_i = \sum_{j=1}^{n} \phi_j [u_i^j + \frac{1}{2} \zeta h_j \; ({}^{t+\Delta t} \hat{e}_{3i}^j - {}^t \hat{e}_{3i}^j)] \tag{3.15} $$

Here ${}^t u_i^j$ and u_i^j denote, respectively, the displacement and incremental displacement components in the x_i-direction at the j-th node. The unit vectors $\hat{\underset{\sim}{e}}_1^i$ and $\hat{\underset{\sim}{e}}_2^i$ can be obtained from the relations

$$\hat{\underset{\sim}{e}}_1^i = (\hat{\underset{\sim}{E}}_2 \times {}^t\hat{\underset{\sim}{e}}_3^i)/|\hat{\underset{\sim}{E}}_2 \times {}^t\hat{\underset{\sim}{e}}_3^i|$$

$$\hat{\underset{\sim}{e}}_2^i = {}^t\hat{\underset{\sim}{e}}_3^i \times {}^t\hat{\underset{\sim}{e}}_1^i \tag{3.16}$$

where $\hat{\underset{\sim}{E}}_2$ is the unit vector along the (global) x_2-axis. If we assume that the angles θ_1^i and θ_2^i are very small, then we can write

$$\hat{\underset{\sim}{e}}_3^i = - {}^t\hat{\underset{\sim}{e}}_2^i\theta_1^i + {}^t\hat{\underset{\sim}{e}}_1^i\theta_2^i \tag{3.17}$$

Substituting Eq. (3.17) into Eq. (3.15), we obtain

$$u_i = \sum_{j=1}^{n} \psi_j[u_i^j + \frac{1}{2}\zeta h_j(- {}^t\hat{e}_{2i}^j\theta_1^j + {}^t\hat{e}_{1i}^j\theta_2^j)] \tag{3.18a}$$

or

$$\{u\} = [T]\{\Delta\} \tag{3.18b}$$

where $\{u\}$ is the column of three displacements at a point, $\{\Delta\}$ is the column of 5n (five per node) displacements: u_i^j, θ_1^j, θ_2^j, $j = 1,2,\ldots,n$; $i = 1,2,3$, and [T] is the transformation matrix defined by Eq. (3.18a). Thus, for each time step one can find the normal vectors from Eq.(3.16) and (3.17) and the incremental displacements at each point from Eq. (3.18), once the five generalized displacements at each node are known. Next we discuss the procedure of determining the generalized displacements of an element.

3.3.3 Element Stiffness Matrix

The strain-displacement equations (3.4) can be expressed in the operator form

$$\{e\} = [A]\{u_o\} \tag{3.19}$$

where $\{e\} = \{e_{11}\ e_{22}\ e_{33}\ 2e_{12}\ 2e_{13}\ 2e_{23}\}^T$, $[A]$ is a function of ${}^t_0u_{i,j}$, and $\{u_o\}$ is the vector of the components of the displacement gradient

$$\{u_o\} = \{u_{1,1}\ u_{1,2}\ u_{1,3}\ u_{2,1}\ u_{2,2}\ u_{2,3}\ u_{3,1}\ u_{3,2}\ u_{3,3}\}^T \tag{3.20}$$

The vector $\{u_o\}$ is related to the displacement increments by

$$\{u_o\} = [N]\{u\} = [N][T]\{\Delta\} \tag{3.20}$$

and

$$\begin{aligned}\{e\} &= [A][N][T]\{\Delta\} \\ &\equiv [B]\{\Delta\}\end{aligned} \tag{3.21}$$

where $[N]$ is the operator of differentials.

Substitution of Eq. (3.21) into Eq. (3.8) yields

$$\int_{V_o} \rho_o [T]^t\{\ddot{u}\}dV_o + ({}^t[K_L] + {}^t[K_{NL}])\{\Delta\} = {}^{t+\Delta t}\{R\} - {}^{t+\Delta t}\{F\} \tag{3.22}$$

where ${}^t[K_L]$, ${}^t[K_{NL}]$, $\{R\}$, and $\{F\}$ are the linear and nonlinear stiffness matrices, force vector, and unbalanced force vectors:

$${}^t[K_L] = \int_{V_o} {}^t[B]^T[C]\ {}^t[B]dV_o \quad , \quad {}^t[K_{NL}] = \int_{V_o} {}^t[B]^T[S]\ {}^t[B]dV_o$$

$$\{F\} = \int_{V_o} {}^t[B]^T\{\hat{S}\}dV_o \tag{3.23}$$

Here $[S]$ and $\{\hat{S}\}$ denote the matrix and vector, respectively, of the second Piola-Kirchhoff stress.

Since we are dealing with laminated composite structures, the important thing is how to perform the integration through the thickness. One way is to pick Gaussian points through the thickness and

then no explicit integration through the thickness is performed. The CPU time will be increased if the number of layers is increased, because the integration should be performed separately for each layer. The other way is to perform explicit integration through the thickness and reduce the integral to a two-dimensional problem. The Jacobian matrix, in general, is a function of ξ_1, ξ_2, and ζ. Zienkiewicz, et al. [35] suggested that terms in ζ to the first power may be neglected, provided the thickness-to-curvature ratios are small. This approximation implies that the derivatives of x_i with respect to ξ_1, ξ_2, and ζ are substantially the same at either end of a mid-surface-normal line. Thus, the Jacobian [J] becomes independent of ζ and explicit integration can be employed. If ζ terms are retained in [J], Gaussian points through the thickness should be added. In the present study, it is assumed that the Jacobian is independent of ζ.

3.3.4 Time Integration and Mass Matrix

Any attempt to solve Eq. (3.22), whether by direct integration or by modal analysis, should take advantage of the symmetry and bandedness of the stiffness matrix. The initial conditions for Eq. (3.22) are the displacements and velocities at time $t = 0$; therefore, the direct integration requires the information at the previous time t, in order to predict the state of motion at the current time $t+\delta t$. The direct integration techniques can be divided into two types: explicit and implicit integrations (see [41-45]). In explicit integration, we solve $\underset{\sim}{u}$ at time $t+\delta t$ based on the equilibrium conditions of the structure at time t. The central difference method is an example of explicit integration method. In the implicit method the solution at time $t+\delta t$ is based on the equilibrium condition of the structure at

time t+Δt. The Houbolt, Wilson, and Newmark methods provide examples of the implicit method. In the present study the Newmark direct integration scheme is employed.

The Newmark integration scheme can be thought of as an extension of the linear acceleration method:

$$ {}^{t+\Delta t}\{\Delta\} = {}^{t}\{\Delta\} + \Delta t^{2}\{\dot{\Delta}\} + [(\tfrac{1}{2} - \beta)\,{}^{t}\{\ddot{\Delta}\} + \beta\,{}^{t+\Delta t}\{\ddot{\Delta}\}](\Delta t)^{2} \quad (3.24) $$

$$ {}^{t+\Delta t}\{\dot{\Delta}\} = {}^{t}\{\dot{\Delta}\} + [(1 - \gamma)\,{}^{t}\{\ddot{\Delta}\} + \gamma\,{}^{t+\Delta t}\{\ddot{\Delta}\}]\Delta t $$

where $\{\Delta\}$ is the generalized displacement vector of any point and β and γ are the dimensionless parameters of generalized acceleration. Chan et al. [46] have discussed the special case $\beta = \frac{1}{12}$ and $\gamma = \frac{1}{2}$, which coincides with a procedure developed by Fox and Goodwin [47]. For the constant average acceleration we have $\beta = \frac{1}{3}$ and $\gamma = \frac{1}{2}$, and for the linear acceleration $\beta = \frac{1}{6}$ and $\gamma = \frac{1}{2}$.

To apply the Newmark integration scheme to the equilibrium equations (3.22), we start from

$$ {}^{t+\Delta t}\{\Delta\}^{(k)} = {}^{t+\Delta t}\{\Delta\}^{(k-1)} + \{\Delta\}^{(k)} \quad (3.25) $$

where k is the iteration number. The velocity and acceleration of any point in the element can be written as

$$ {}^{t}\dot{u}_i = \sum_{j=1}^{n} \phi_j \, {}^{t}\dot{u}_i^{j} + \sum_{j=1}^{n} \frac{1}{2}\phi_j \zeta h_j ({}^{t}\dot{\hat{e}}_{3i}^{j} - {}^{o}\dot{\hat{e}}_{3i}^{j}) \quad (3.26) $$

$$ {}^{t}\ddot{u}_i = \sum_{j=1}^{n} \phi_j \, {}^{t}\ddot{u}_j^i + \sum_{i=1}^{n} \frac{1}{2} \phi_j \zeta h_j ({}^{t}\ddot{\hat{e}}_{3i}^{j} - {}^{0}\ddot{\hat{e}}_{3i}^{j}) \qquad (3.27) $$

Since we are dealing with transient problems, the load vector $\{R\}$ can be a function of time. From Eq. (3.27) we have for the i-th component

$$ {}^{t+\Delta t}\ddot{\Delta}_i = a_0 ({}^{t+\Delta t}\Delta_i - {}^{t}\Delta_i) - a_2 {}^{t}\dot{\Delta}_i - a_3 {}^{t}\ddot{\Delta}_i $$

$$ {}^{t+\Delta t}\dot{\Delta}_i = {}^{t}\dot{\Delta}_i + a_4 {}^{t}\ddot{\Delta}_i + a_5 {}^{t+\Delta t}\ddot{\Delta}_i \qquad (3.28) $$

where

$$ a_0 = \frac{1}{\beta(\Delta t)^2} \quad , \quad a_1 = \frac{\gamma}{\beta \Delta t} \quad , \quad a_2 = \frac{1}{\beta \Delta t} $$

$$ a_3 = \frac{1}{2\beta} - 1 \quad , \quad a_4 = \Delta t(1 - \gamma) \quad , \quad a_5 = \gamma \Delta t \qquad (3.29) $$

Substituting Eq. (3.22) into Eq.(3.28), we obtain

$$ {}^{t+\Delta t}\ddot{\hat{e}}_{3i}^{j} = a_0 (\theta_1^j \hat{e}_{2i}^j - \theta_2^j \hat{e}_{1i}^j) - a_2 \dot{\hat{e}}_{3i}^j - a_3 {}^{t}\ddot{\hat{e}}_i^j + {}^{t}\ddot{\hat{e}}_{3i}^j $$

$$ {}^{t+\Delta t}\dot{\hat{e}}_{3i}^{i} = {}^{t}\dot{\hat{e}}_{3i}^{j} + a_4 {}^{t}\ddot{\hat{e}}_{3i}^{j} + a_5 {}^{t+\Delta t}\ddot{\hat{e}}_{3i}^{j} \qquad (3.30) $$

Finally the complete approximation of the equations of motion over an element becomes

$$ (a_0 {}^{t}[M] + {}^{t}[K])\{\Delta^{(k)}\} = {}^{t+\Delta t}\{R\} - {}^{t+\Delta t}\{F^{(k-1)}\} $$

$$+ a_2[{}^t\{P_1\} - \frac{1}{\Delta t}({}^t\{P_2\} - {}^t\{P_3\})] + a_3\{P_4\} \qquad (3.31)$$

where

$$[M] = \int_{V_o} \rho_o \, {}^t[T]^T \, {}^t[T] \, dV_o$$

$$\{P_1\} = \int_{V_o} \rho_o \, {}^t\{\dot{\Delta}\} \, [T] dV_o$$

$$\{P_2\} = \int_{V_o} \rho_o \, {}^{t+\Delta t}\{\Delta\}^{(k-1)} [T] dV_o$$

$$\{P_3\} = \int_{V_o} \rho_o \, {}^t\{\Delta\} [T] dV_o$$

$$\{P_4\} = \int_{V_o} \rho_o \, {}^t\{\ddot{\Delta}\} \, [T] dV_o \qquad (3.32)$$

Equation (3.31) can be expressed in the following final form:

$$[\hat{K}]\{\Delta\} = {}^{t+\Delta t}\{\hat{R}\} - {}^{t+\Delta t}\{F\}^{(k-1)} \qquad (3.33)$$

where

$$[\hat{K}] = a_o \, {}^t[M] + {}^t[K]$$

$${}^{t+\Delta t}\{\hat{R}\} = {}^{t+\Delta t}\{R\} + a_2 \, [{}^t\{P_1\} - \frac{1}{\Delta t}({}^t\{P_2\} - {}^t\{P_3\})] + a_3 \, \{P_4\} \qquad (3.34)$$

This completes the finite-element formulation of the 3-D degenerated element.

CHAPTER IV

NUMERICAL VALIDATION OF THE ELEMENTS

4.1 Introduction

The present chapter is devoted to the validation of the finite elements developed herein based on the two-dimensional shell theory and three-dimensional continuum theory. The elements are validated by comparing the present results with those available in the literature for static bending, natural vibration, and transient response of isotropic plates and shells. Numerical results are presented to bring out the limitations and restrictions of the present elements. All of the results presented here were obtained on an IBM 370/3081 computer with double precision arithmetic.

The results to be discussed are grouped into three major categories: (1) static bending, (2) natural vibration, and (3) transient response. All results, except for the vibrations, are presented in a graphical form.

4.2 Static Analysis

Here we present a discussion of four example problems, all involving shell structures.

4.2.1 Cylindrical Panel Under the Influence of Gravity (i.e., under its own weight)

Consider the circular cylindrical panel shown in Fig. 4.1. The geometric parameters and material properties are listed below:

$R = 25.0$ ft	$E = 3 \times 10^6$ psi
$a = 25.0$ ft	$\nu = 0.0$
$h = 3.0$ in.	$g = 90$ psi
$\theta = 40°$	

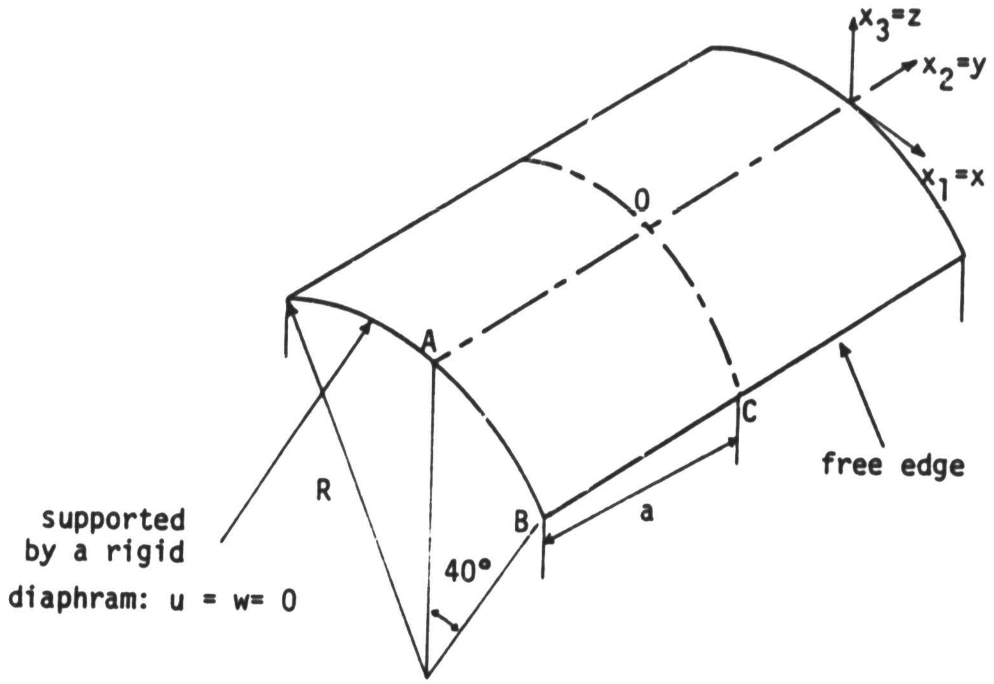

Figure 4.1 Geometry of the cylindrical shell used in Problem 1 of Section 4.1

The panel is supported on rigid diaphragms on the curved edges and free on the straight edges. We wish to find the deformation of the panel under its own weight. Many authors have employed this problem as a standard test problem for checking new elements or numerical schemes. The radial displacements along the central curved line are shown in Fig. 4.2 for both the 3-D element and the 2-D deformable shell element. The longitudinal displacements along the supported edges are shown in Fig. 4.3. A uniform mesh of 2 x 2 eight-node elements was used in one quadrant for this analysis. The solutions agree very closely with those obtained in Reference [48].

4.2.2 Cylindrical Shell Subjected to Radial Pressure

Consider the circular cylindrical panel shown in Fig. 4.4. The shell is clamped along four edges and subjected to uniform radial inward pressure. The loading is nonconservative, that is, the direction of the applied load is normal to the cylindrical surface at any time during the deformation. The geometric and material properties are

$$R = 2540 \text{ mm}, \ a = b = 254 \text{ mm}, \ h = 3.175 \text{ mm},$$

$$\theta = 0.1 \text{ rad}, \ E = 3.10275 \text{ kN/mm}^2, \ \nu = 0.3$$

Due to the symmetry of the geometry and deformation, only one quarter of the panel is analyzed. A load step of 0.5 KN/m^2 was used in order to get a close representation of the deformation path. Fig. 4.5 shows the central deflection versus the pressure for the first of the three sets of panel dimensions:

(i) a = 254 mm, b = 254 mm (deep shell)

(ii) a = 635 mm, b = 635 mm

(iii) a = 1270 mm, b = 1270 mm (shallow shell)

In cases (ii) and (iii) the analysis was limited to linear solutions.

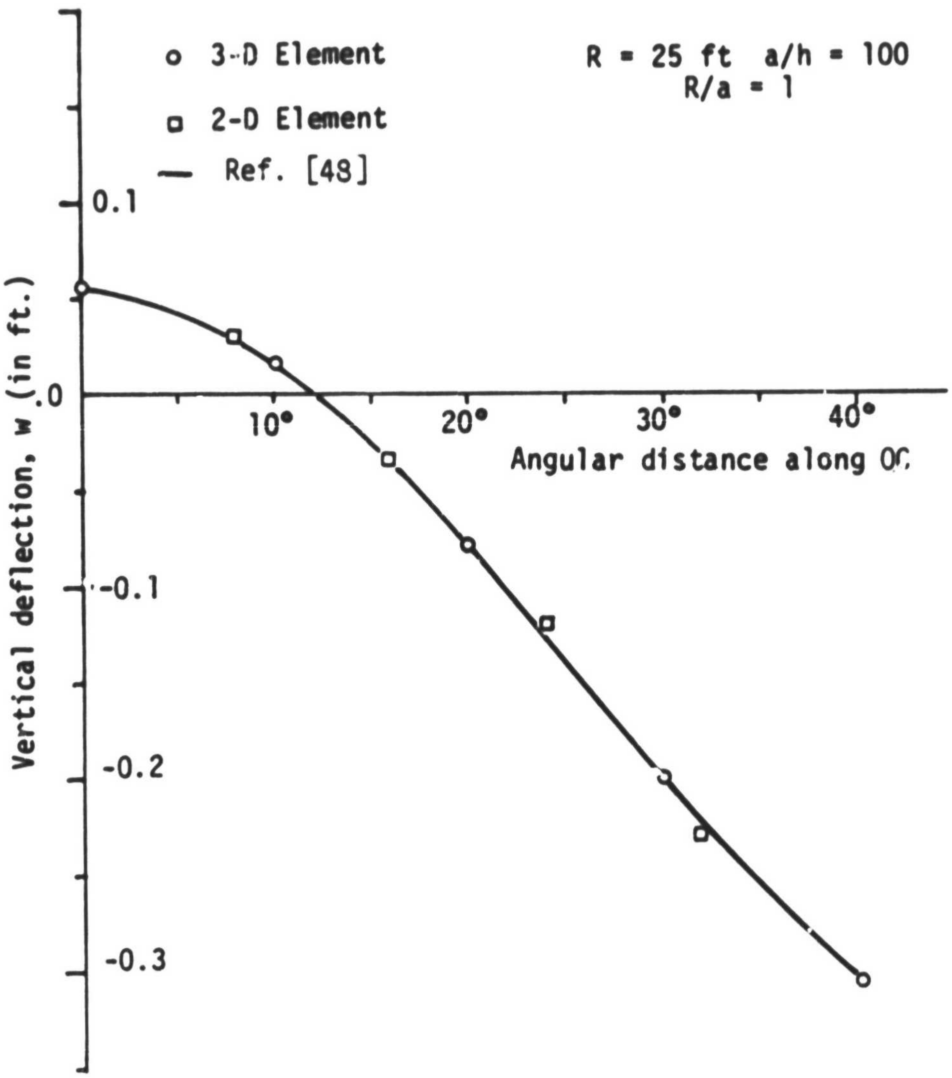

Figure 4.2 Vertical deflection along the midsection OC (see Fig. 4.1 for the geometry)

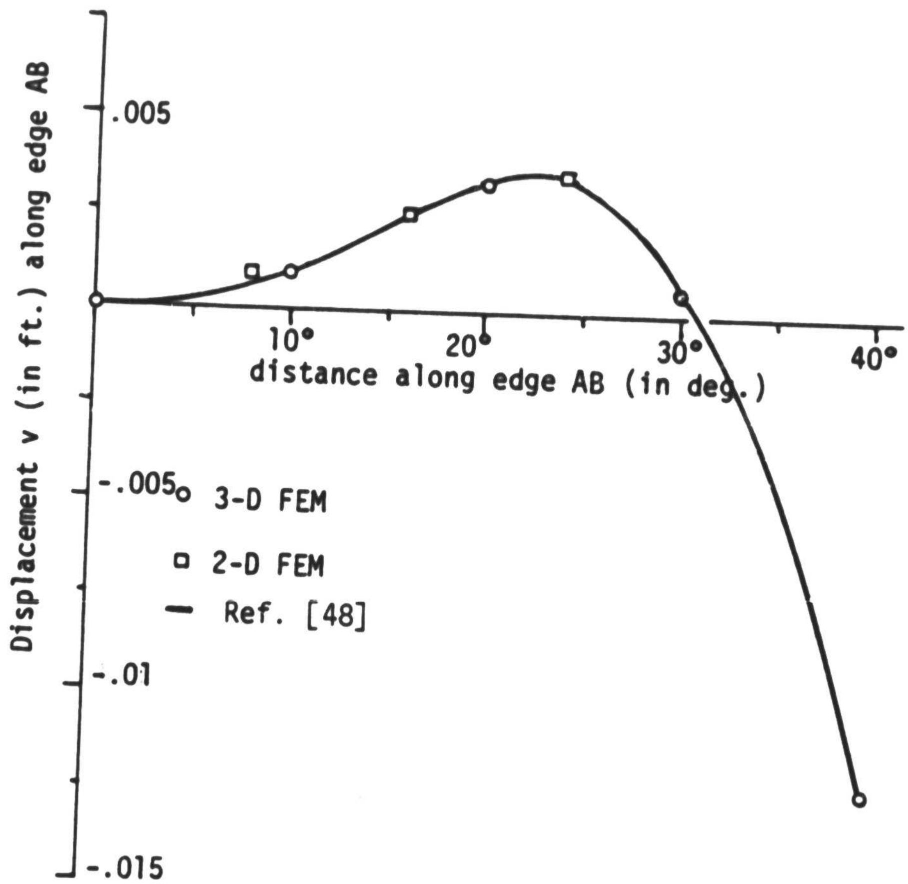

Figure 4.3 Displacement v along the supported edge AB

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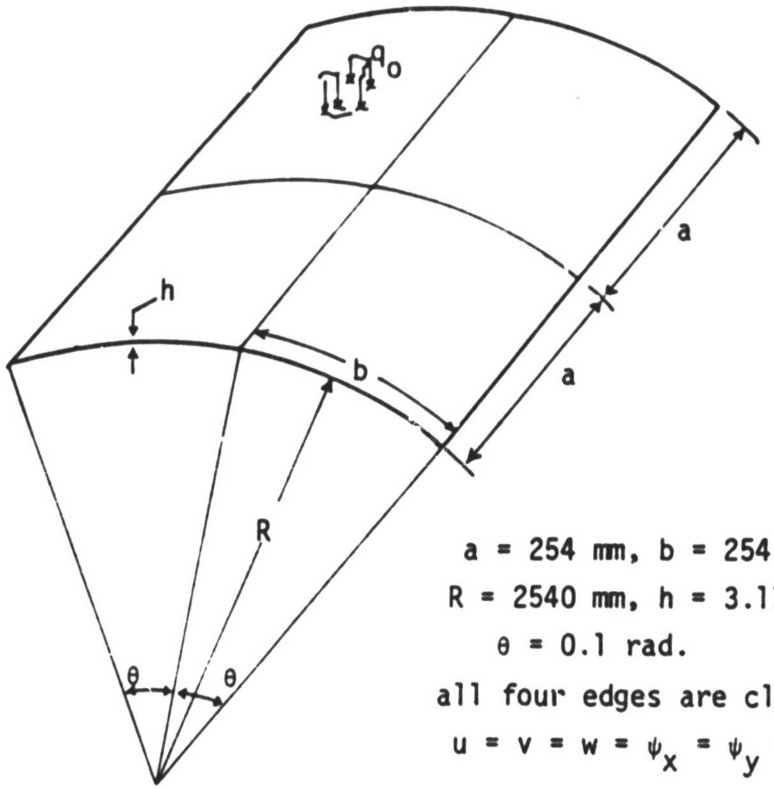


a = 254 mm, b = 254 mm,
R = 2540 mm, h = 3.175 mm,
θ = 0.1 rad.
all four edges are clamped:
$u = v = w = \psi_x = \psi_y = 0$

Figure 4.4 Geometry of the cylindrical shell problem discussed in Section 4.2.2.

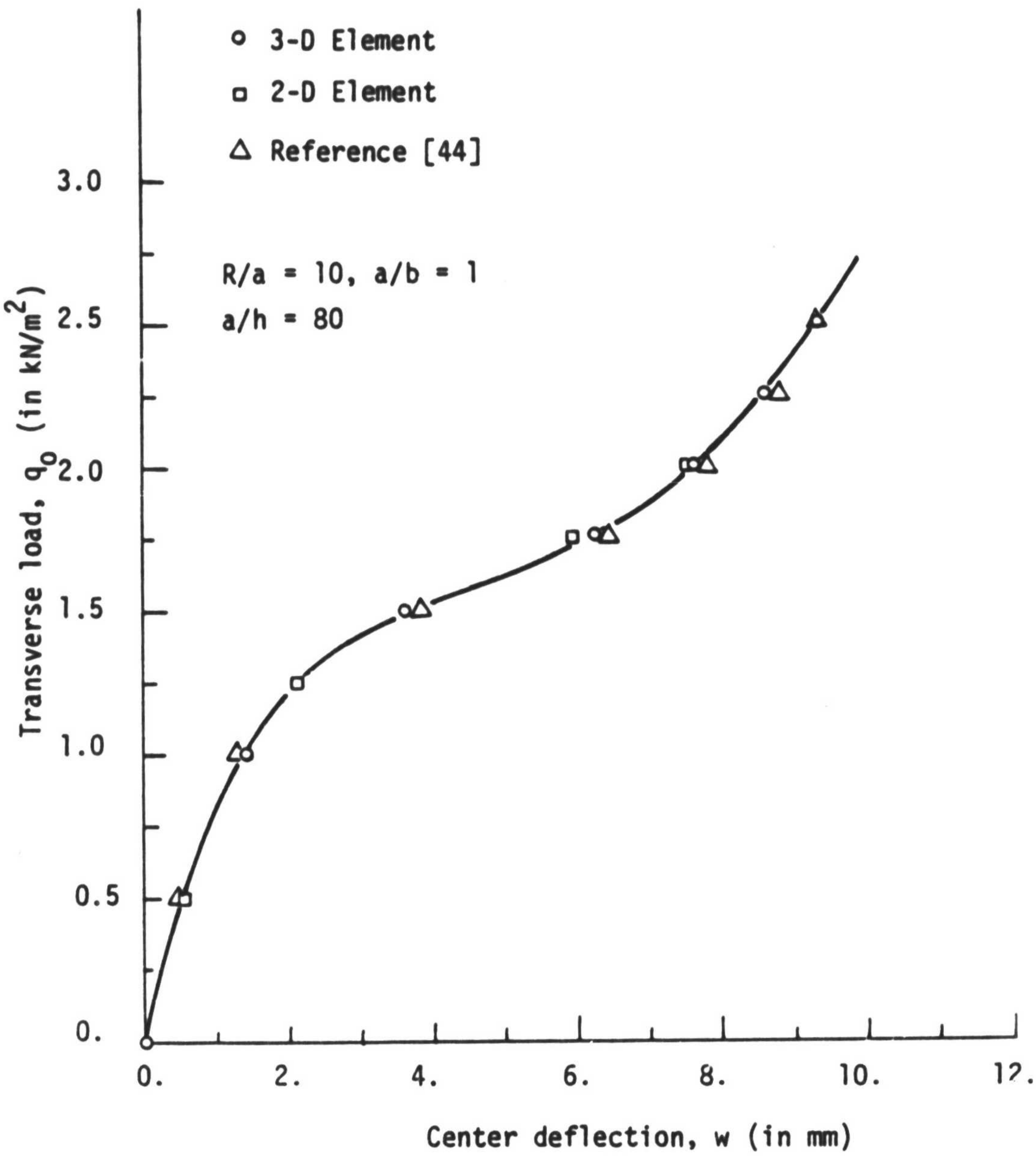


Figure 4.5 Load-deflection curve for the clamped cylindrical shell problem discussed in Section 4.2.2

Table 4.1 contains a comparison of the center deflections obtained by using the two elements for various meshes. From the results presented in the table it is clear that for shallow shells both elements give almost the same results. The difference in the deflections predicted by the two elements increases as the shell becomes shallow (case (iii)).

4.2.3 Cylindrical Shell Subjected to Center Point Load

Figure 4.6 shows a circular cylindrical shell which has the same geometric (except, h = 12.7 mm) and material properties as the one in Problem 2. The longitudinal boundaries are hinged and immovable, and the curved edges are free. A concentrated point load is applied at the center. One quarter of the panel was analyzed using a 2x2 mesh of nine-node quadrilateral elements. The load step is 0.5kN. Figure 4.7 contains the plot of the central deflection versus the load. The results agree very closely with those obtained by Dhatt [49].

4.2.4 Spherical Shell Subjected to Point Load at the Center

The spherical shell shown in Fig. 4.8 is subjected to a concentrated load P at the crown. The boundaries are all hinged and immovable. The geometric parameters and material properties are given below:

$$R_1 = R_2 = 2540 \text{ mm}$$

$$a = b = 784.9 \text{ mm}$$

$$h = 99.45 \text{ mm}$$

$$E = 68.95 \text{ N/mm}$$

$$\nu = 0.3$$

One quarter of the shell was analyzed by a 2x2 mesh of nine node quadrilateral elements. The load step is 10 kN. Figure 4.9 contains a

Table 4.1 Comparison of linear center deflections obtained by the 2-D and 3-D elements for Problem 2.

Case	Mesh	3-D	2-D
(i)	2x2	0.52579	0.52321
	3x3	0.52505	0.52265
(ii)	2x2	0.35888	0.39431
	3x3	0.3205	0.34228
(iii)	2x2	0.30636	0.37667
	3x3	0.28674	0.27021

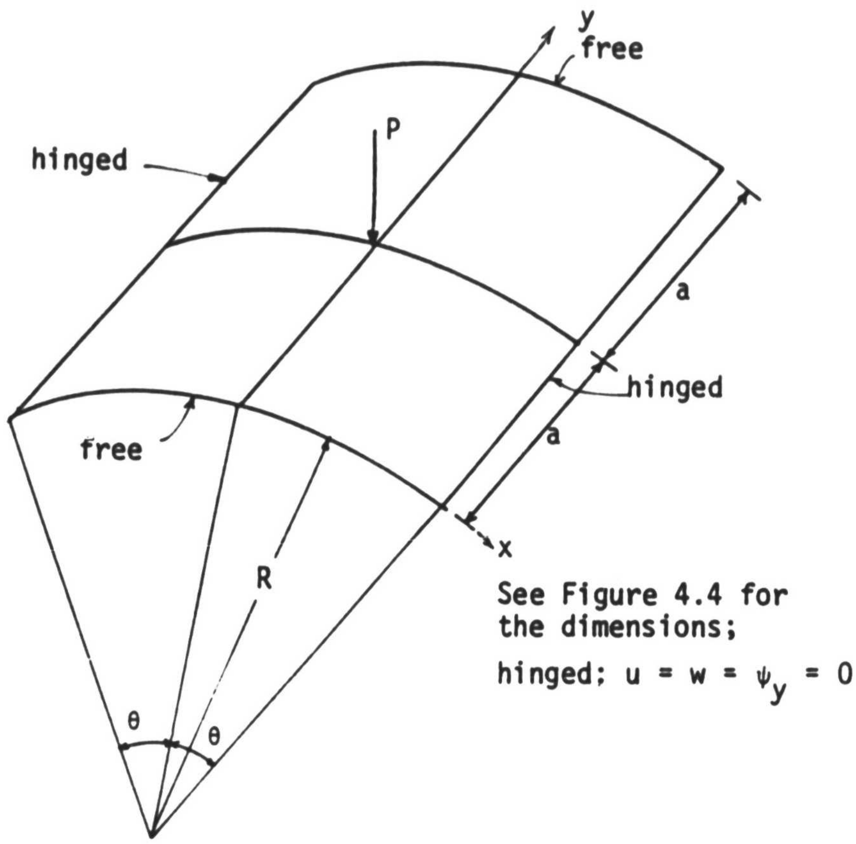


Figure 4.6 Geometry of the cylindrical shell problem discussed in Section 4.2.3

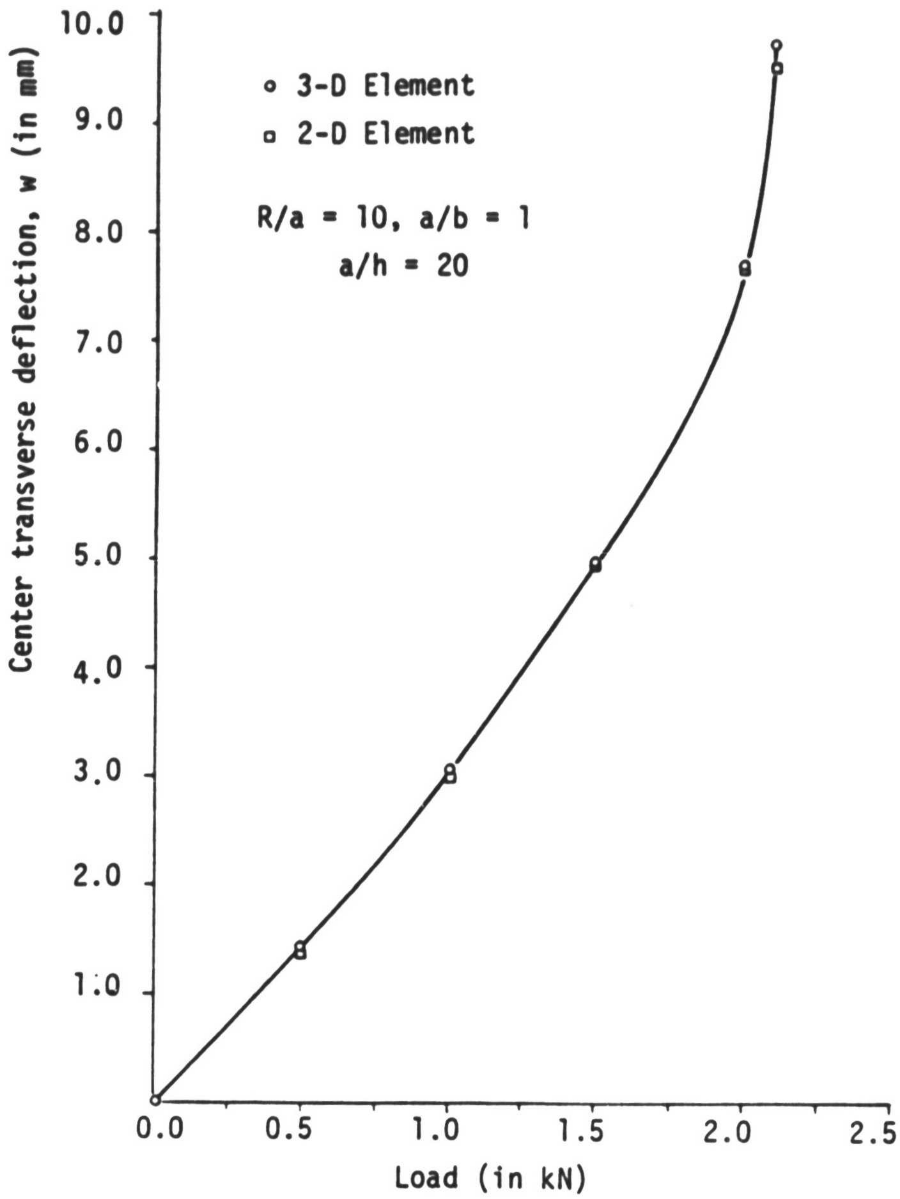
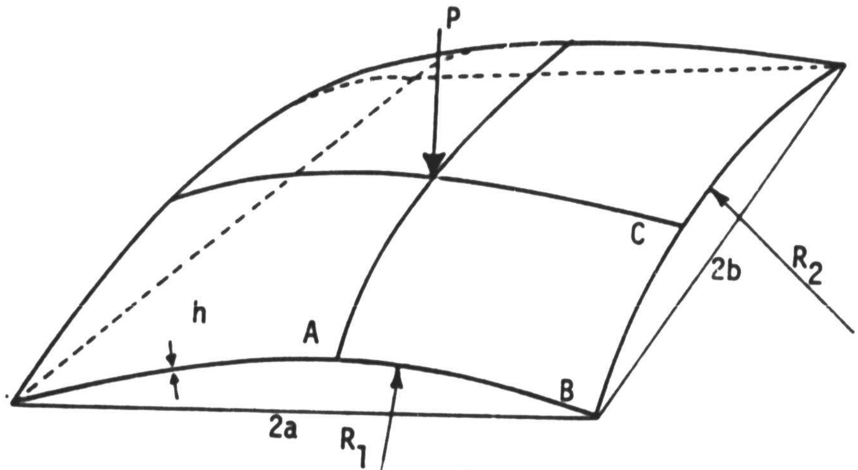


Figure 4.7 Load-deflection curve for the cylindrical shell problem discussed in Section 4.2.3



all edges are hinged:

$u = w = \psi_y = 0$ on BC

$v = w = \psi_x = 0$ on AB

$R_1 = R_2 = 2540$ mm,

$a = b = 784.9$ mm,

$h = 99.45$ mm.

Figure 4.8 Geometry of the spherical shell discussed in Section 4.2.4.

comparison of the center deflections obtained using the 2-D and 3-D elements, which agree with that of Leicester [50], who used a shallow shell 2-D finite element.

4.3 Natural Vibration of Cantilevered Twisted Plates

Here we discuss the results obtained for natural frequencies of cantilevered twisted plates. This analysis was motivated by their relevance to natural vibrations of turbine blades. Consider a cantilevered plate with a twist angle θ at the free end. The plate is made of an isotropic material. Table 4.2 contains the natural frequencies of a square plate for various values of the twist angle θ. A 2x2 mesh and 4x4 mesh of nine-node elements are employed to study the convergence trend. The results of the refined mesh are included in the parentheses. The results agree with many others published in a recent NASA report. Tables 4.2-4.5 contain natural frequencies of twisted plates for various aspect ratios and side-to-thickness ratios.

4.4 Transient Analysis

Here we present results of the nonlinear transient analysis of a cantilevered beam and a spherical shell. Both problems have been solved by other investigators.

4.4.1 Cantilevered Beam Under Uniformly Distributed Load

Consider a cantilever beam under a uniformly distributed transverse step load, as shown in Fig. 4.10. The geometric parameters and material properties are given below:

$$L = 10 \text{ in}$$

$$h = 1 \text{ in}$$

$$b = 1 \text{ in}$$

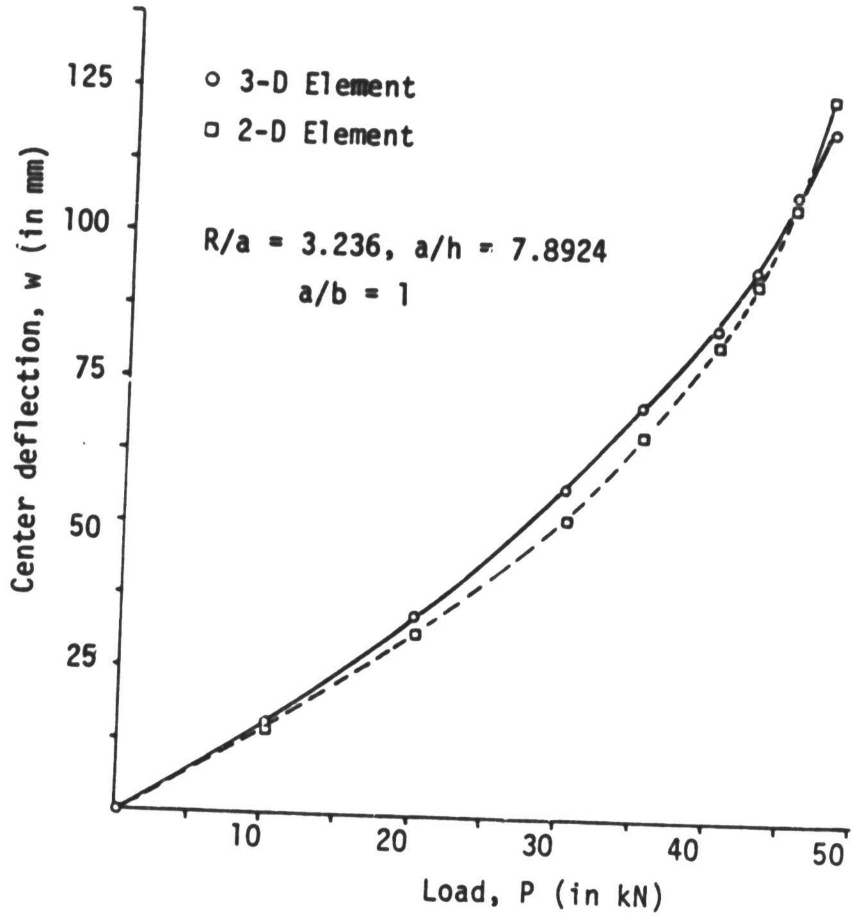


Figure 4.9 Load-deflection curve for the spherical shell problem discussed in Section 4.2.4

Table 4.2 Natural Frequencies of Twisted Plate Vibration (a/b = 1, a/h = 20)

$$\bar{\omega} = \omega a^2\sqrt{\rho h/D} \quad , \quad D = \frac{Eh^3}{12(1-\nu^2)} \quad , \quad \nu = 0.3$$

Twist Angle	Mode 1	2	3	4	5	6
0°	* 3.4556 **(3.4583)	8.4110 (8.3353)	22.0999 (21.0238)	28.2089 (26.7465)	31.9740 (30.1454)	55.1625 (52.0784)
15°	3.4359	10.2920	21.5199	27.2054	32.7430	44.5375
30°	3.3790 (3.3694)	13.7014 (14.2222)	19.9840 (18.9795)	25.0943 (26.8104)	34.3341 (34.4591)	45.8987 (45.7547)
45°	3.2908	18.1009	15.9097	23.5680	35.5332	45.7013
60°	6.1800	17.8319	15.5635	24.1842	36.1466	44.9152

* 2 x 2,9-node mesh
**4 x 4,9-node mesh

Table 4.3 Natural Frequencies of Twisted Plate Vibration (b/a = 3, a/h = 20, 3 x 3, 9-node mesh)

$$\bar{\omega} = \omega b^2\sqrt{\rho h/D} \quad , \quad D = \frac{Eh^3}{12(1-\nu^2)} \quad , \quad \nu = 0.3$$

Twist Angle	Mode 1	2	3	4	5	6	7
0°	3.4150	20.8772	21.6190	65.9706	66.2590		127.256
15°	3.4009	20.8798	22.1118	21.6032	68.0938	69.3258	130.284
30°	3.3598	19.4048	25.3743	60.2183	73.5180	77.4493	138.176
45°	3.2956	17.5289	29.8404	58.2600	80.9488	88.5246	148.8975
60°	3.2136	15.7431	34.8827	55.8921	89.2028	100.7760	155.070

Table 4.4 Natural Frequencies of Twisted Plate Vibration (a/b = 1, a/h = 5)

$$\bar{\omega} = \omega b^2 \sqrt{\rho h / D} \quad , \quad D = \frac{Eh^3}{12(1-\nu^2)} \quad , \quad \nu = 0.3$$

Twist Angle	Mode 1	2	3	4	5	6
0°	* 3.33916 **(3.3390)	7.3948 (7.3559)	10.8083 (10.883)	18.4930 (17.757)	23.7907 (22.769)	26.0552 (24.125)
15°	3.31713 (3.3170)	7.4816 (7.4504)	10.8053 (10.774)	18.4043 (17.771)	23.6767 (22.694)	24.9474 (24.083)
30°	3.2538 (3.2538)	7.7593 (7.7089)	10.5248 (10.478)	18.4091 (17.795)	23.3734 (22.471)	24.6116 (23.943)
45°	3.1570 (3.1569)	8.1435 (8.0728)	10.1270 (10.062)	18.3843 (17.79)	22.9126 (22.117)	24.0566 (23.651)
60°	3.0370 (3.0366)	8.5855 (8.4814)	9.67198 (8.5911)	18.3089 (17.730)	22.3670 (21.684)	23.3533 (23.160)

* 2 x 2,9-node mesh
**3 x 3,9-node mesh

Table 4.5 Natural Frequencies of Twisted Plate Vibration (b/a = 3, a/h = 5, 3 x 3,9-node mesh)

$$\bar{\omega} = \omega b^2 \sqrt{\rho h/D} \quad , \quad D = \frac{Eh^3}{12(1-\nu^2)} \quad , \quad \nu = 0.3$$

Twist angle	Mode 1	2	3	4	5	6
0°	3.3908	15.551	19.124	21.065	59.924	61.949
15°	3.3161	15.192	19.231	21.572	60.088	60.830
30°	3.3336	14.379	19.549	22.811	60.576	58.472
45°	3.2674	13.449	20.060	24.404	61.360	55.874
60°	3.1833	12.548	20.741	26.139	62.416	53.381

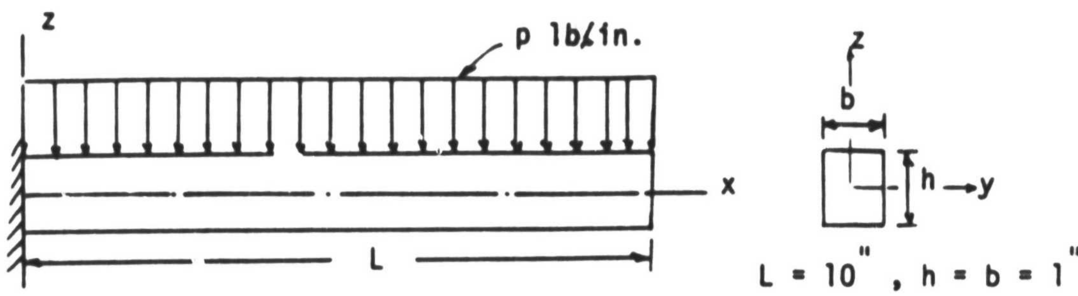


Figure 4.10 Cantilevered beam under uniformly distributed load

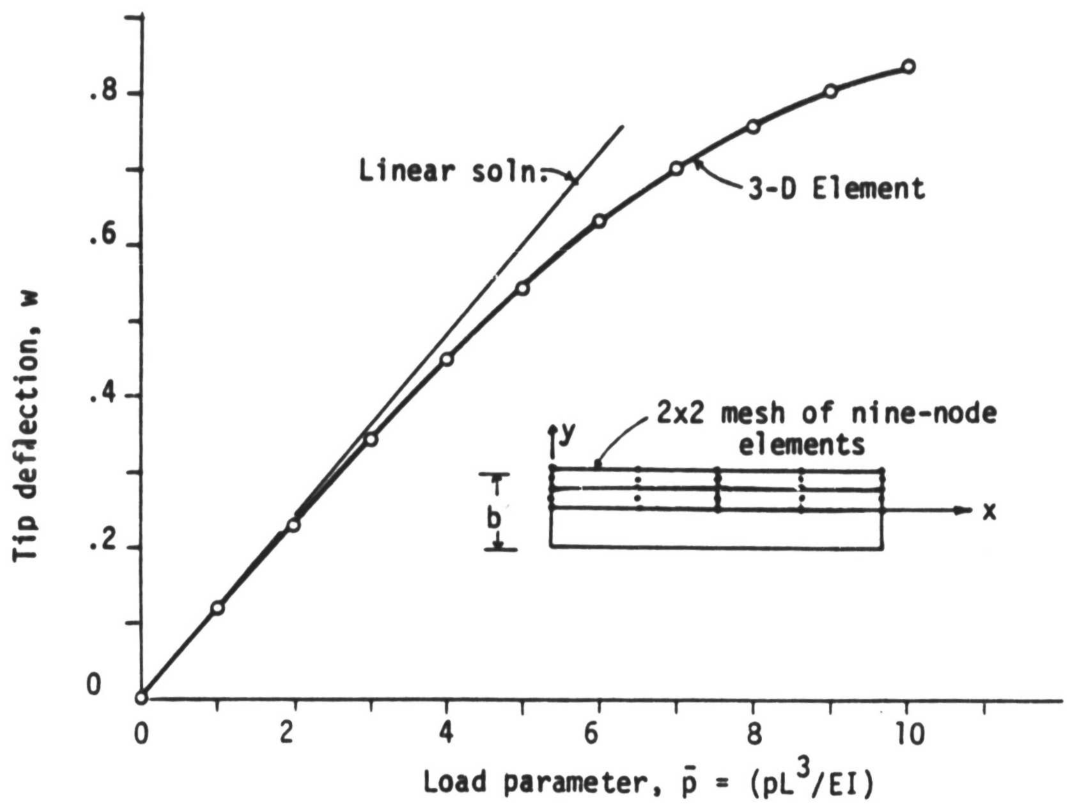


Figure 4.11 Load-deflection curve for the static bending of the cantilevered beam shown in Figure 4.10

$$E = 1.2 \times 10^6 \text{ psi}$$

$$\nu = 0.2$$

$$\rho = 10 \text{ lb sec}^2/\text{in}^4$$

The cantilever is analyzed using four nine-node 2-D shell and 3-D degenerated elements (see Fig. 4.11). The load applied is nonconservative, because the load follows the deformed beam and stays normal to it at all times. Figure 4.11 shows the tip deflection versus load obtained in the static analysis. Figure 4.12 contains plots of tip deflection versus time; obtained using the 2-D and 3-D elements and by Bathe et al. [37]. The time step employed is 1.35×10^{-4} sec. The reason for the difference between the solutions predicted by the shell element and 3-D element is that the load in the 3-D element follows the deformed shape and is perpendicular to the deformed beam, whereas in the 2-D shell theory it is always vertical. Consequently, the vertical load component is larger in the 2-D shell element than in the 3-D element, and thus explains the difference in the solutions.

4.4.2 Spherical Cap Under Axisymmetric Pressure Loading

Consider a spherical cap, clamped on the boundary and subjected to axisymmetric pressure loading. The geometric and material properties are

$$R = 22.27 \text{ in}$$

$$h = 0.41 \text{ m}$$

$$E = 10.5 \times 10^6 \text{ psi}$$

$$\nu = 0.3$$

$$\rho g = 0.095 \text{ lb/in}^3$$

$$\theta = 26.67^0$$

$$p_0 = 100 \text{ psi}$$

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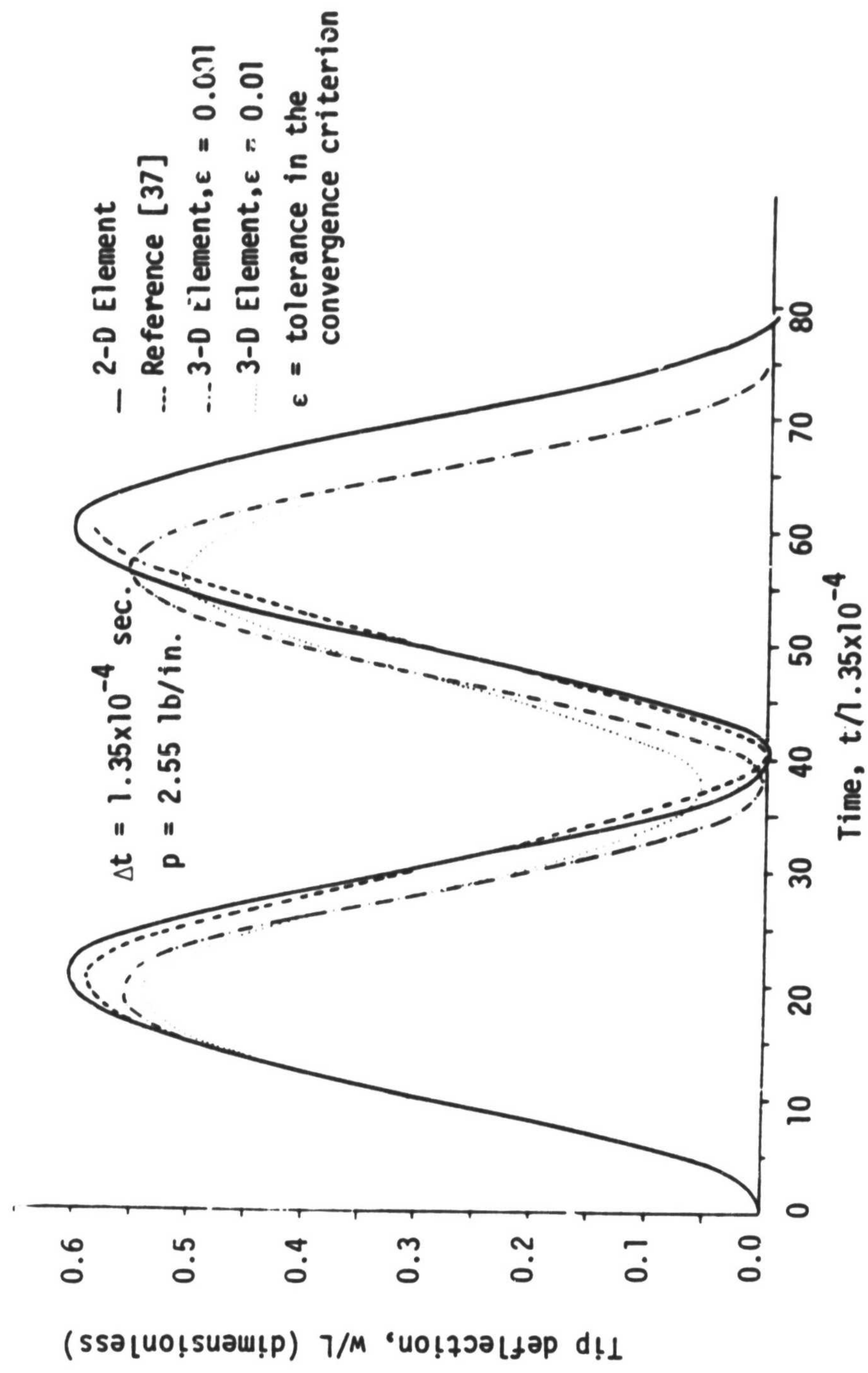

Figure 4.12 Nonlinear deflection at the tip of the cantilevered beam (transverse deflection vs. time)

$$\Delta t = 10^{-5} \text{ sec}$$

This problem has been analyzed by Stricklin, et al. [51] using an axisymmetric shell element. In the present study the spherical cap is discretized into five nine-node 2-D and 3-D elements. Fig. 4.13 shows the center deflection versus time. The present solutions are in excellent agreement in most places with that of Stricklin et al. [51]. The difference between the solutions is mostly in the regions of local minima and maxima.

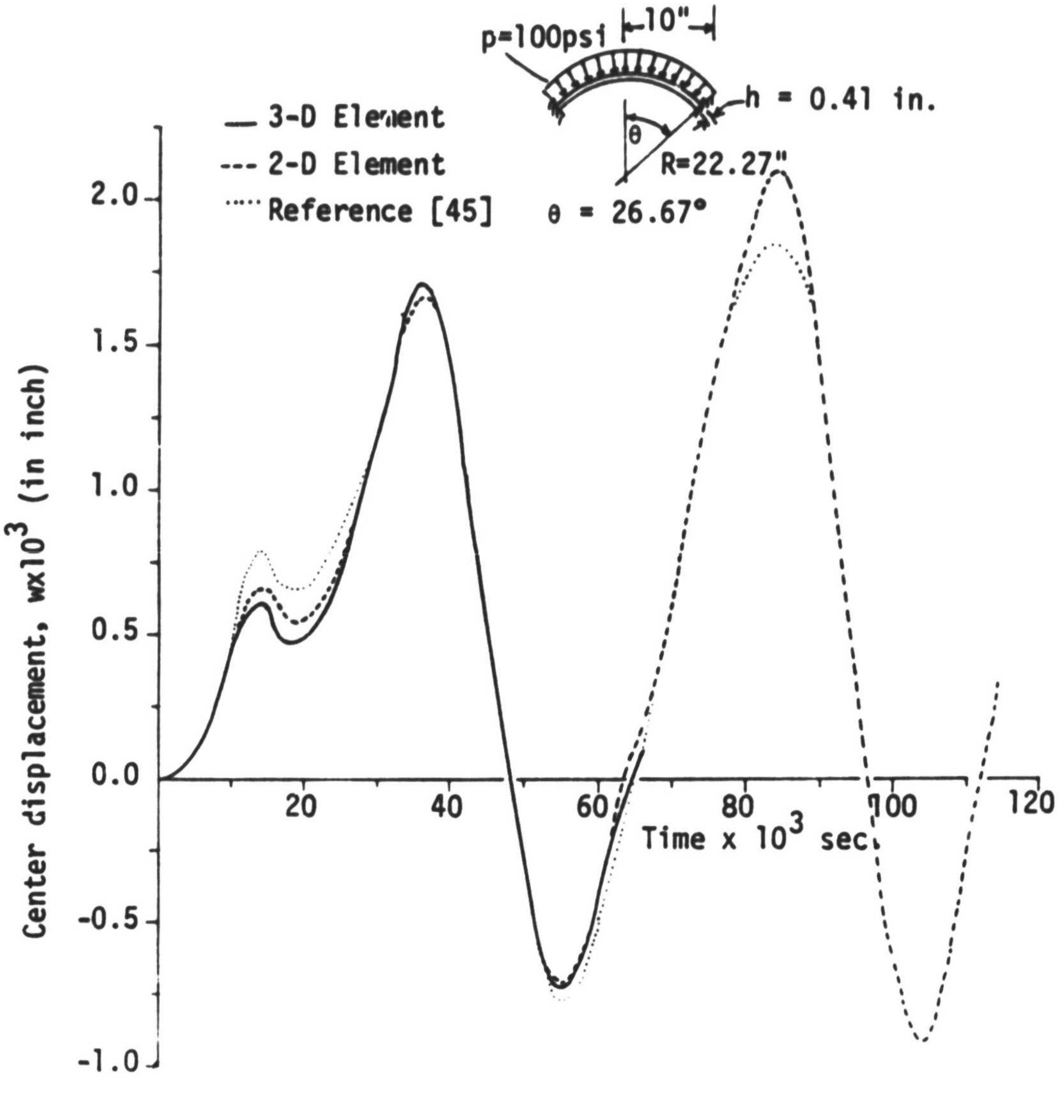

Figure 4.13 Center transverse displacement versus time for a spherical cap under axisymmetric dynamic loading (load = 100 psi.)

CHAPTER V

NUMERICAL RESULTS FOR COMPOSITE PLATES AND SHELLS

5.1 Introduction

In this chapter we discuss the numerical results obtained by the 2-D and 3-D elements for the nonlinear analysis of layered anisotropic composite plates and shells. Most of the results presented here are new and therefore cannot be compared with other results to make quantitative judgements concerning their accuracy. Results for both static bending and transient response are discussed.

5.2 Static Analysis

5.2.1 Orthotropic Cylinder Subjected to Internal Pressure

Consider a clamped orthotropic ($E_2 = 2.0 \times 10^6$ psi, $E_1/E_2 = 3.75$, $G_{12}/E_2 = 0.625$, $\nu = 0.25$) cylinder of radius R = 20 in and length 20 in, and subjected to internal pressure $p_o = 6.41\ \pi$ psi (see Fig. 5.1). The problem was analyzed, for linear deflections only, by Rao [21] using shallow thin shell elements. A mesh of 2x2 nine-node elements is used in an attempt to analyze the problem. The linear center deflections obtained by the 2-D and 3-D elements are 0.0003764 in., and 0.0003739 in., respectively. These values compare favorably with 0.000366 in. of Rao [21] and 0.000367 of Timoshenko's analytical solution [52]. The latter two solutions are based on classical shell theory.

In the large-deflection analysis the present results are compared with those of Chang and Sawamiphakdi [30]. A value of 2.5 ksi is used for the load step. Figure 5.2 shows a comparison of the present deflection with that of [30], which used a 3-D degenerated element based on an updated Lagrangian approach. The agreement is very good.

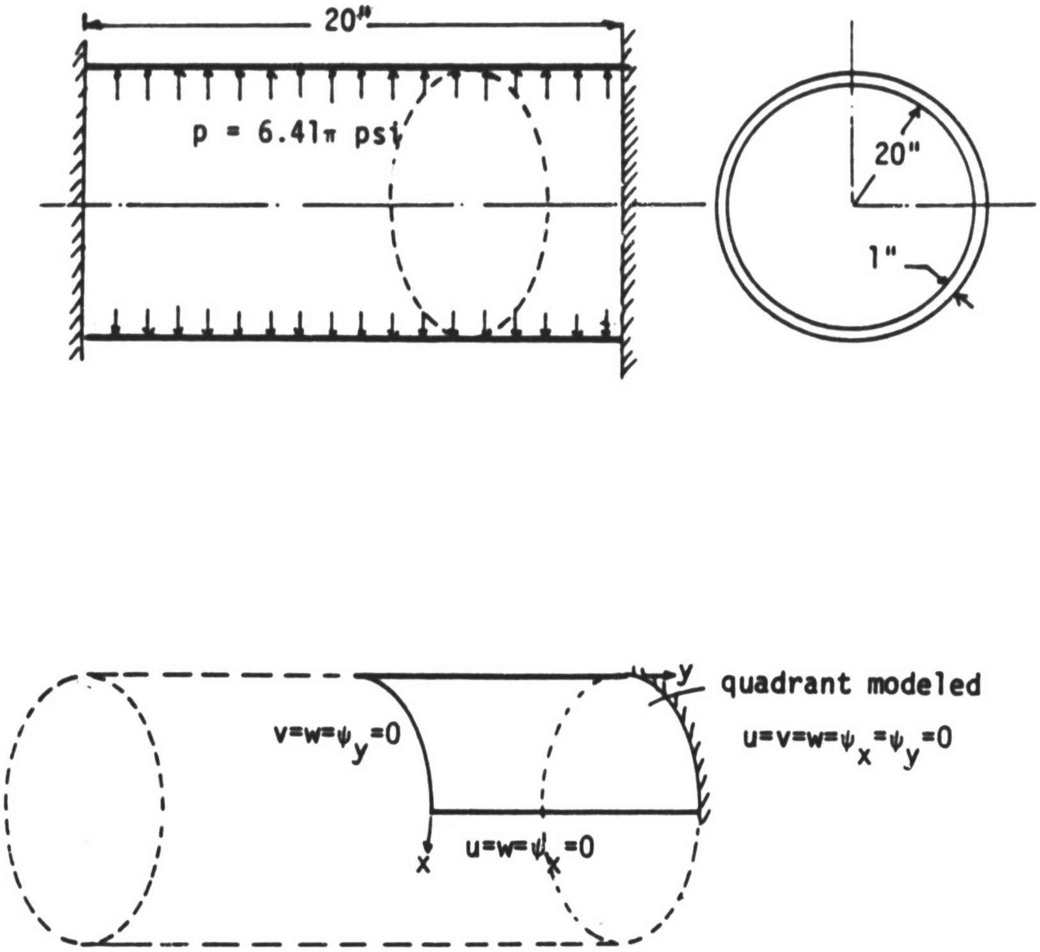

Figure 5.1 Geometry of the cylindrical shell problem discussed in Section 5.2.1

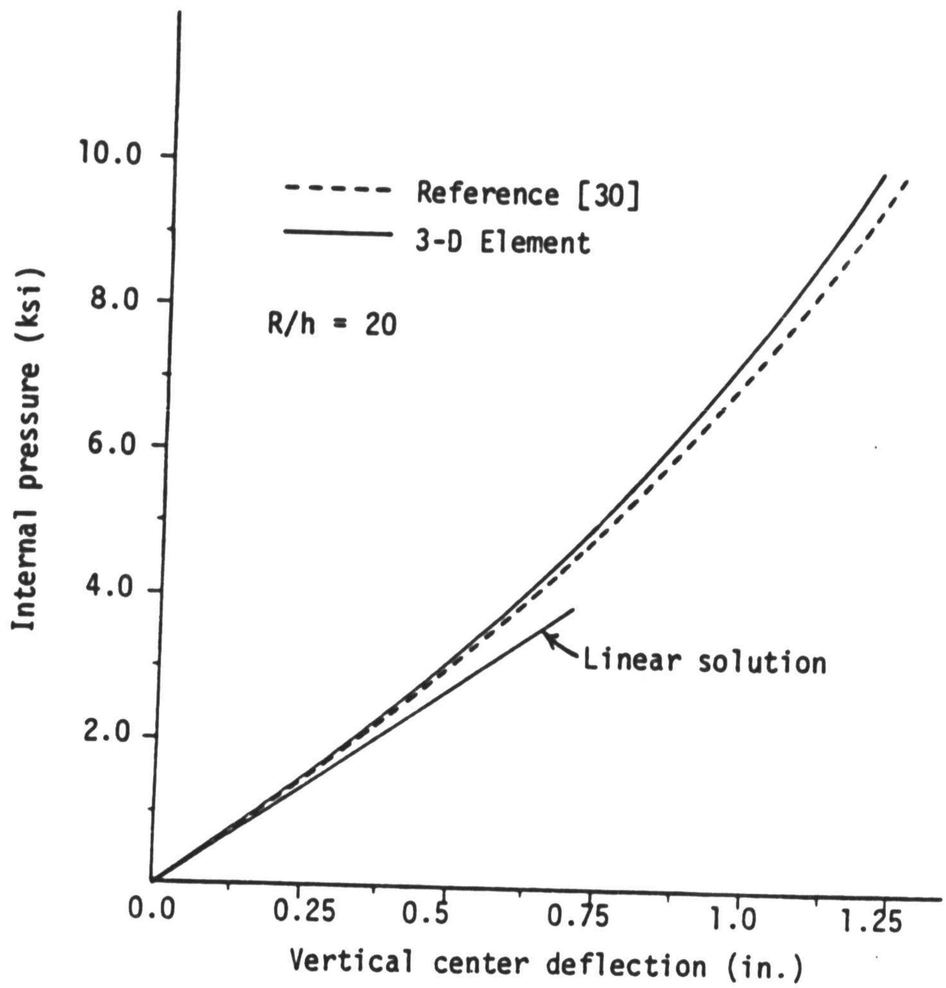

Figure 5.2 Center transverse deflection versus internal pressure

5.2.2 Nine-Layer Cross-Ply Spherical Shell Subjected to Uniform Loading

Consider a spherical shell cross-ply laminated of nine layers of graphite-epoxy material ($E_1/E_2 = 40$, $G_{23}/E_2 = 0.6$, $G_{13} = G_{12} = 0.5\ E_2$, and $\nu_{12} = 0.3$), subjected to uniformly distributed loading, and simply supported on all its edges (i.e., transverse deflection and tangential rotations are zero). In Fig. 5.3, a comparison of the load-deflection curves obtained by the present elements with those obtained by Noor and Hartley [29] is presented (for the parameters $h/a = 0.01$ and $R/a = 10$). The results agree very well with each other, the present 2-D results being closer to Noor and Hartley's solution. This is expected because their element is based on a shell theory.

5.2.3 Two-Layer Cross-Ply and Angle-Ply (45°/-45°) Cylindrical Shells Under Uniform Loading

The geometry of the circular cylindrical shell used here is the same as that shown in Fig. 4.1. The shell is assumed to be simply supported on all its edges. The material properties of individual lamina are the same as those used in Problem 2 of this chapter. A mesh of 2x2 nine-node elements in a quarter shell is used to model the problem. The results of the analysis are presented in the form of load-deflection curves in Fig. 5.4. From the results, one can conclude that the angle-ply shell is stiffer than the cross-ply shell. This can be due to the bending-stretching coupling.

5.2.4 Two-Layer Cross-Ply and Angle-Ply (45°/-45°) Spherical Shells Under Uniform Loading

The geometry and boundary conditions used in this problem are the same as those used in Problem 2 of this chapter. The geometric parameters used are: $R/a = 10$, $a/h = 100$. The load-deflection curves for the cross-ply and angle-ply shells are shown in Fig. 5.5. From the

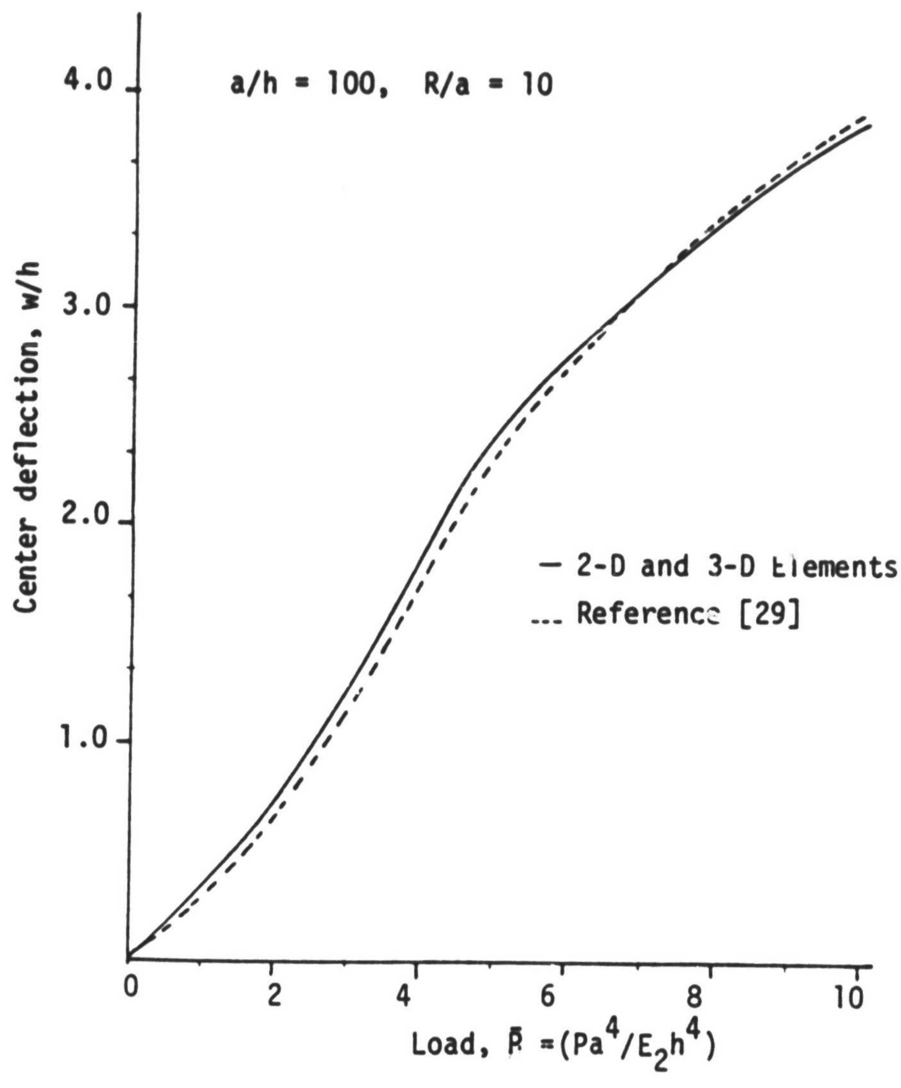

Figure 5.3 Deflection versus load parameter for nine-layer cross-ply (0°/90°/0°/...) spherical shell discussed in Section 5.2.2.

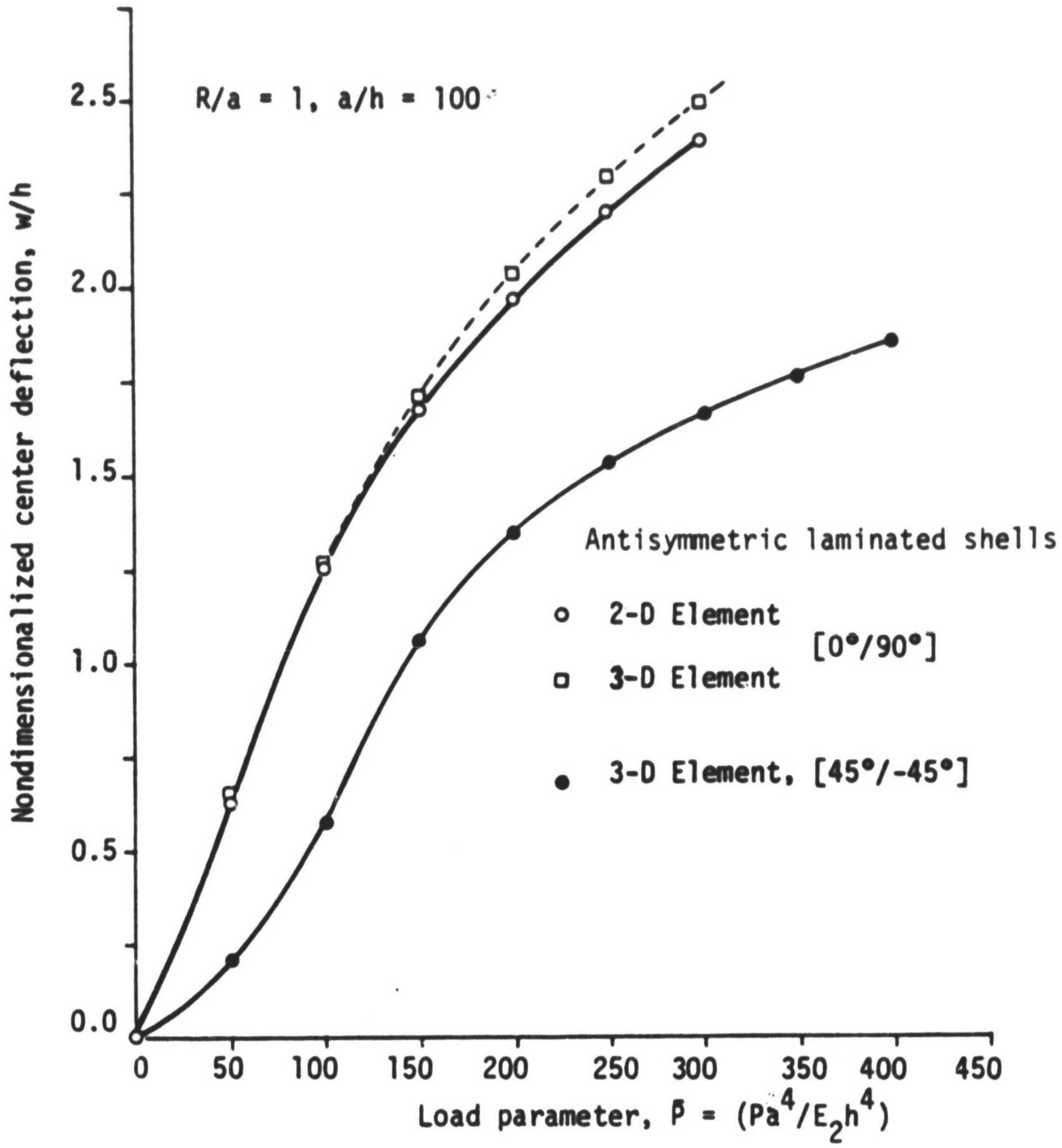

Figure 5.4 Deflection versus the load parameter for two-layer composite cylindrical shell (see Figure 4.1)

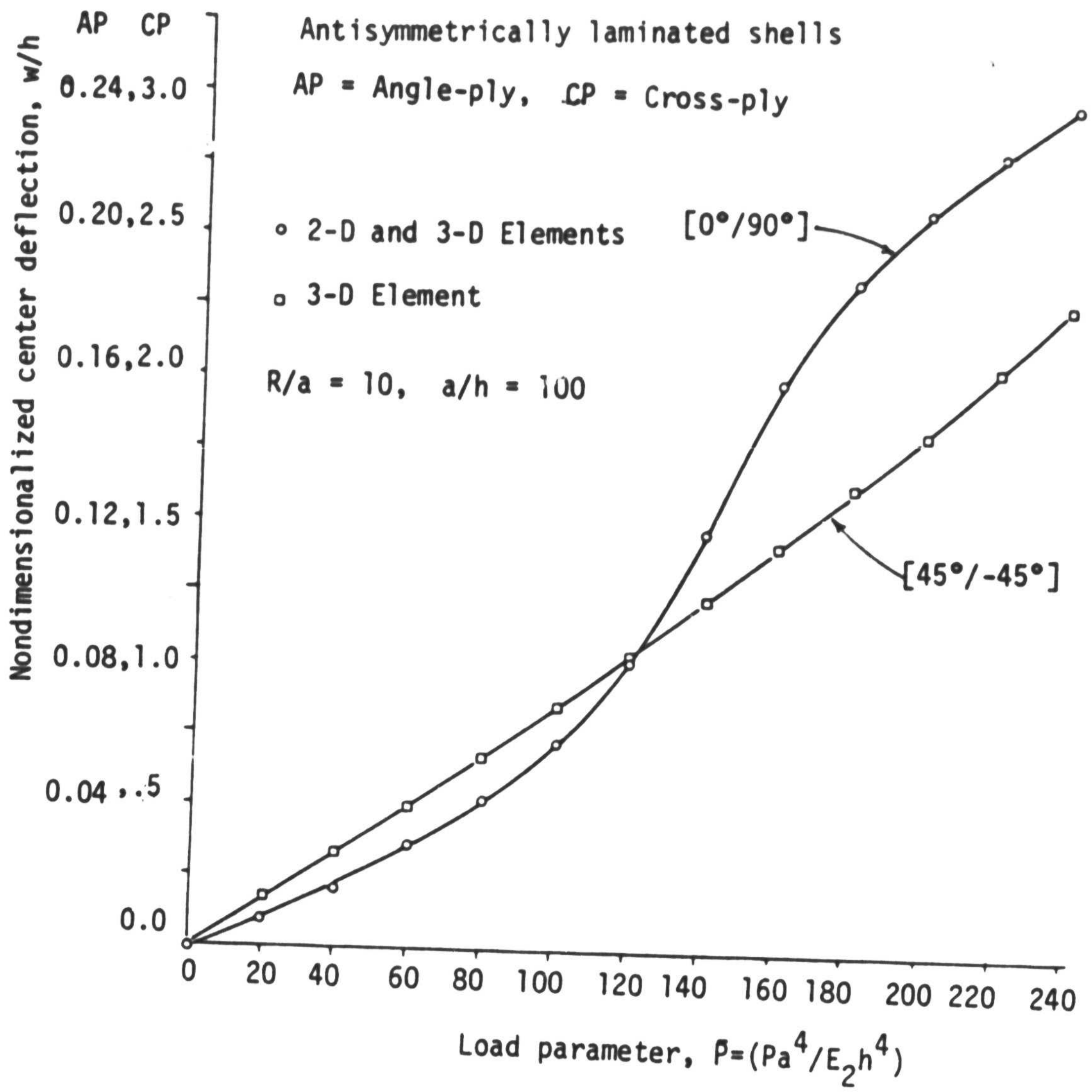

Figure 5.5 Nondimensionalized deflection versus the load for laminated shells discussed in Section 5.2.4.

plot it is apparent that, for the load range considered, the angle-ply shell, being stiffer, does not exhibit much geometric nonlinearity. The load-deflection curve of the cross-ply shell exhibits a varying degree of nonlinearity with the load. For load values between 100 and 150, the shell becomes relatively more flexible. (which can be due to bending-stretching coupling)

5.3 Transient Analysis

5.3.1 Two-Layer Cross-Ply Plate Under Uniform Load

The problem is the same as that analyzed by Reddy [53] using a plate element. The geometric and material parameters used are

$$a = b = 244 \text{ in.}, \; h = 0.635 \text{ in.}, \; P_0 = 0.5 \times 10^{-2} \text{ psi}$$

$$E_1 = 17.578 \times 10^6 \text{ psi}, \; E_2 = 0.7031 \times 10^6 \text{ psi}, \; G_{12} = 0.5 \times 10^6 \text{ psi}$$

$$\rho = 0.2547 \times 10^{-5} \text{ lb-sec}^2/\text{in}^4, \; \nu_{12} = 0.25, \; \delta t = 0.002 \text{ sec.}$$

Figure 5.6 contains a plot of the center deflection versus time obtained by the 3-D element. The solution is in excellent agreement with that of Reddy [53].

5.3.2 Two-Layer Cross-Ply Cylindrical Shell Under Uniform Load

A cylindrical shell with $a = b = 5$ in, $R = 10$ in, $h = 0.1$ in is simply-supported on the four edges. The deep shell is laminated with two layers (0°/90°) and loaded by a uniform step load $\hat{P} = \frac{a^4 p}{E_2 h^4} = 50$. Figure 5.7 contains a plot of the center deflection versus time for 2-D and 3-D elements with $\delta t = 0.1 \times 10^{-4}$ sec. The solutions obtained using the two elements are in good agreement. In Fig. 5.8, the solid line indicates the center deflection versus time for load $\hat{P} = 1000$ and time step $\delta t = 0.3 \times 10^{-5}$ sec. The amplitude is almost twelve times that due to load $\hat{P} = 50$, whereas the load increases twenty times. The dotted

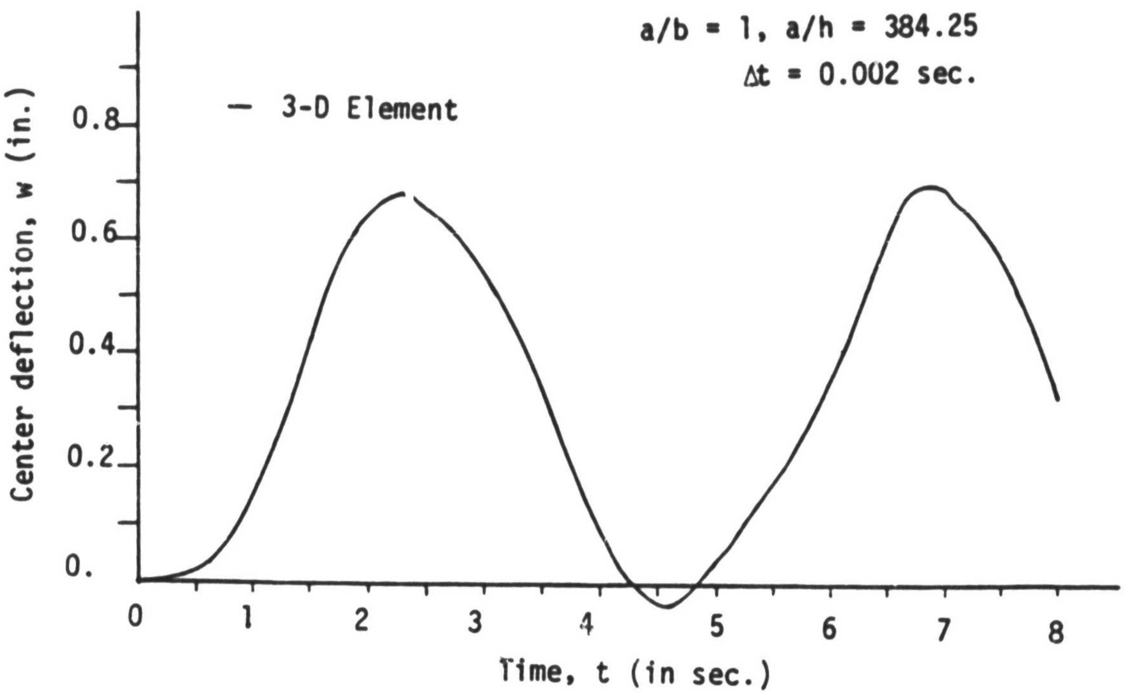

Figure 5.6 Deflection versus time for two-layer plate [0°/90°] under uniformly distributed step load.

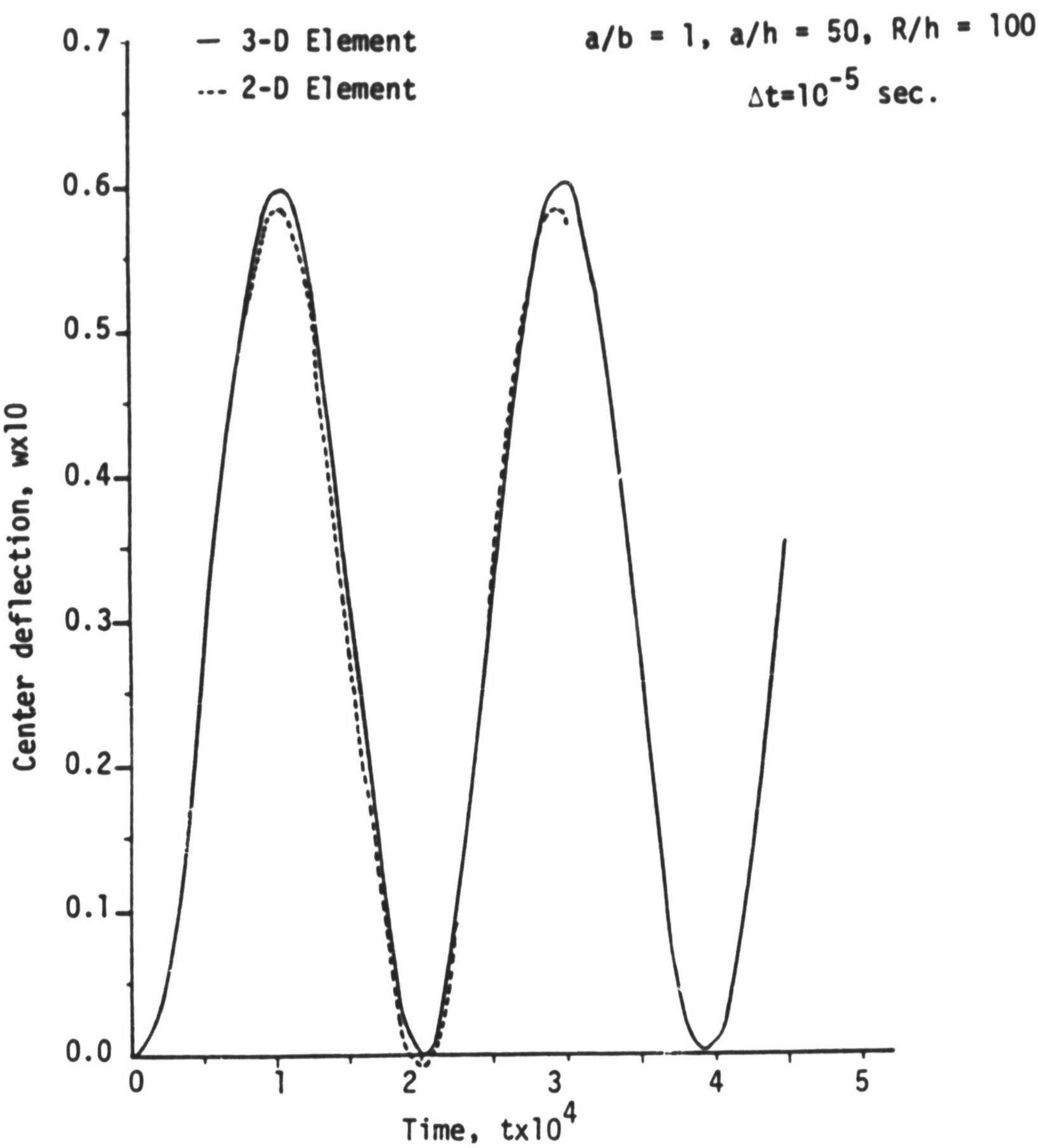

Figure 5.7 Center deflection versus time for two-layer cross-ply cylindrical shell subjected to uniform step load

line in Fig. 5.8 is for $\Delta t = 0.1 \times 10^{-5}$ sec, which gives slightly smaller deflections.

5.3.3 Four-Layer Angle Ply (45°/-45°/45°/-45°) Cylindrical Shell Under Uniform Load

Here we present results for a cylindrical shell which has the same geometry as problem 5.3.2. The four-layer angle ply (45°/-45°/45°/-45°) laminated shell is simply supported on four edges and is subjected to a uniform step load $\hat{P} = 50$. Fig. 5.9 contains a plot of the center deflection versus time for 2-D and 3-D elements. The two elements yield solutions that agree very well, except that the 2-D element gives negative values of the deflection at the end of the cycle. The discrepancy is due to the fact that the 2-D element does not account for geometric changes from one time step to the next.

5.3.4 Two-Layer Angle-Ply (45°/-45°) Spherical Shell Under Uniform Loading

Consider a spherical shell with $a = b = 10$ in, $R = 20$ in and $h = 0.1$ in, simply supported at four edges and excited by a uniform step load. The shell consists of two layers (45°/-45°). Figure 5.10 shows the center deflection versus time for $\hat{P} = 50$ and $\hat{P} = 500$ with time step 0.2×10^{-5} sec. For the small load the curve is relatively smooth compared to that of the larger load. This is due to the fact that the geometric nonlinearity exhibited at $\hat{P} = 50$ is smaller compared to that at $\hat{P} = 500$.

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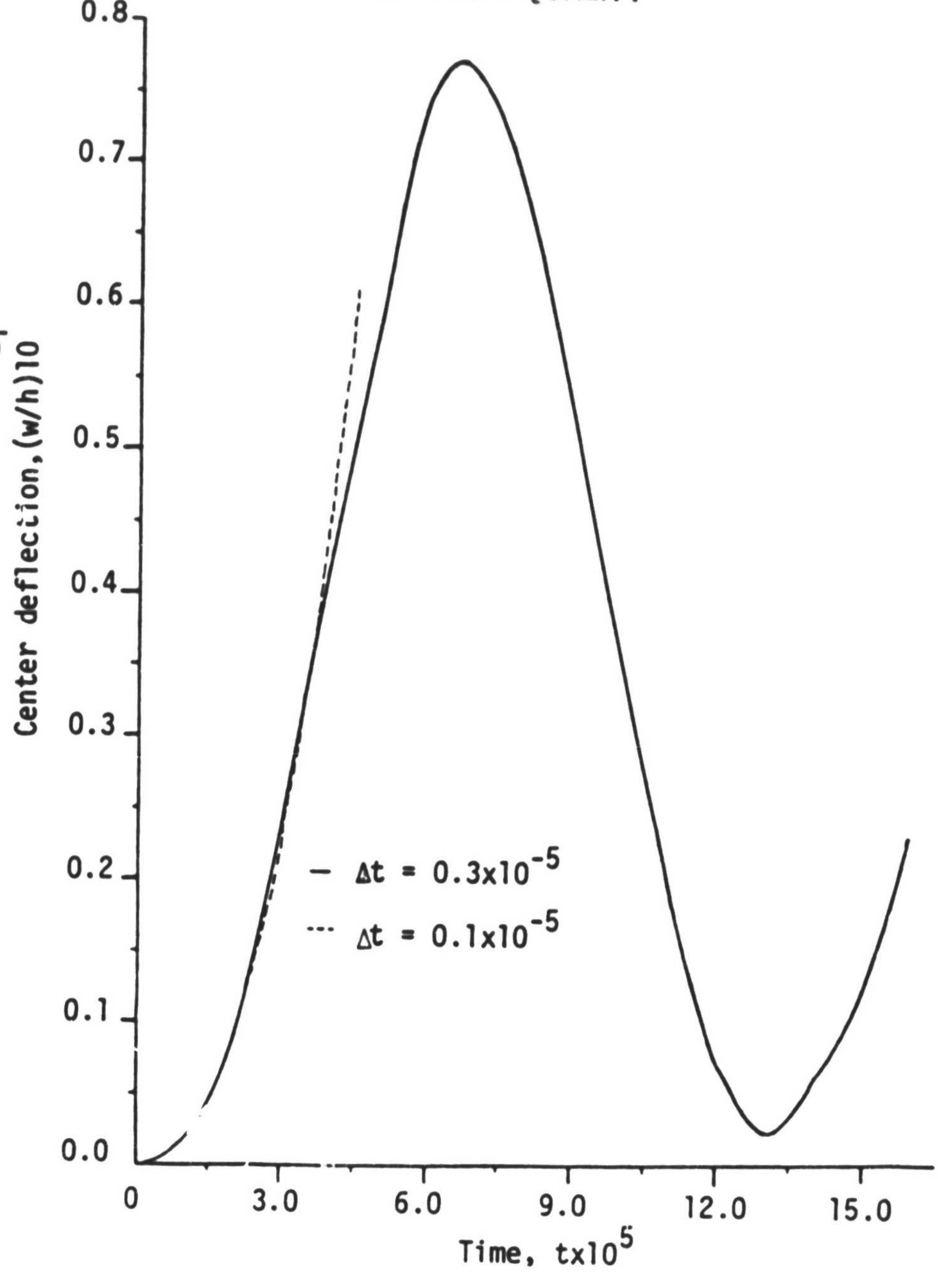

Figure 5.8 Comparison of the center deflection obtained by two different time steps for the problem discussed in Section 5.3.2.

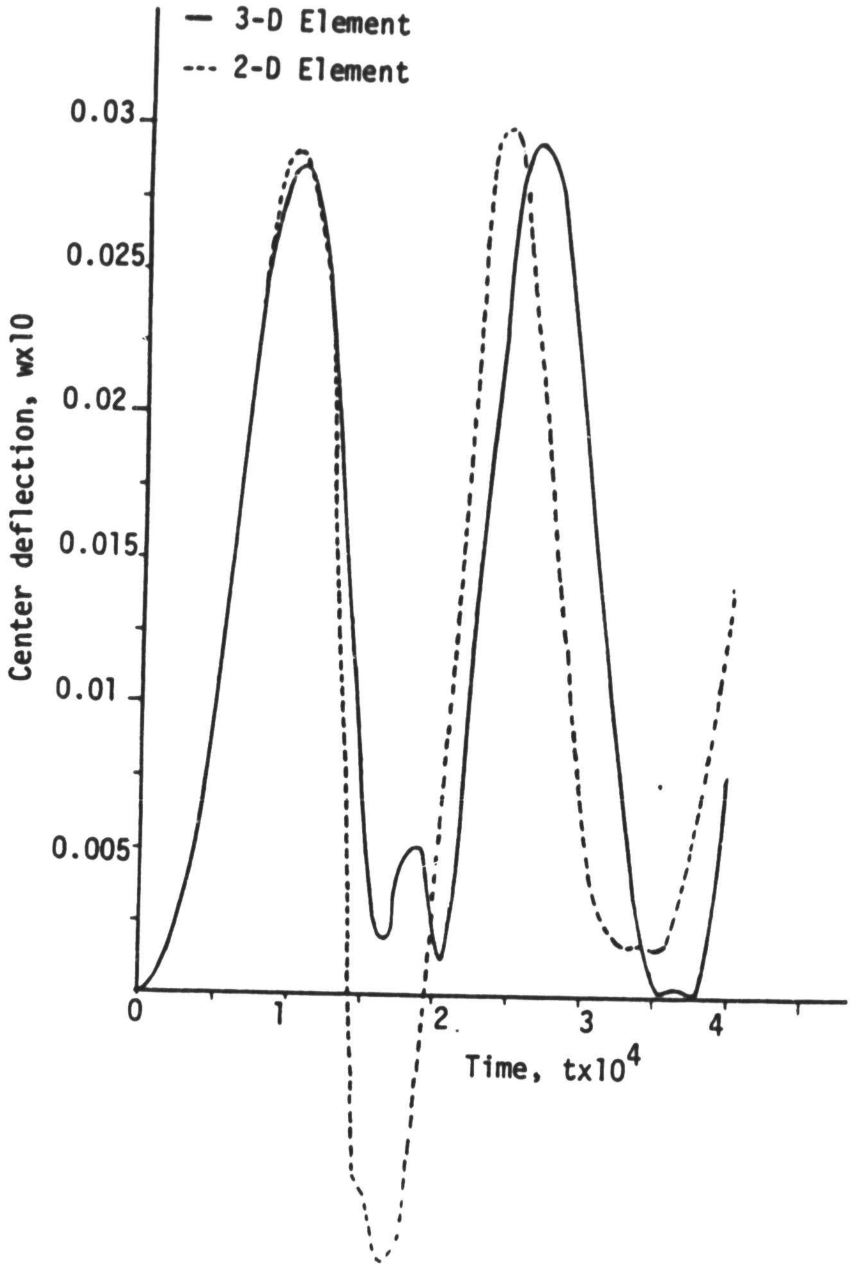

Figure 5.9 Center deflection versus time for four-layer angle-ply [45°/-45°/45°/-45°] cylindrical shell subjected to uniformly distributed load.

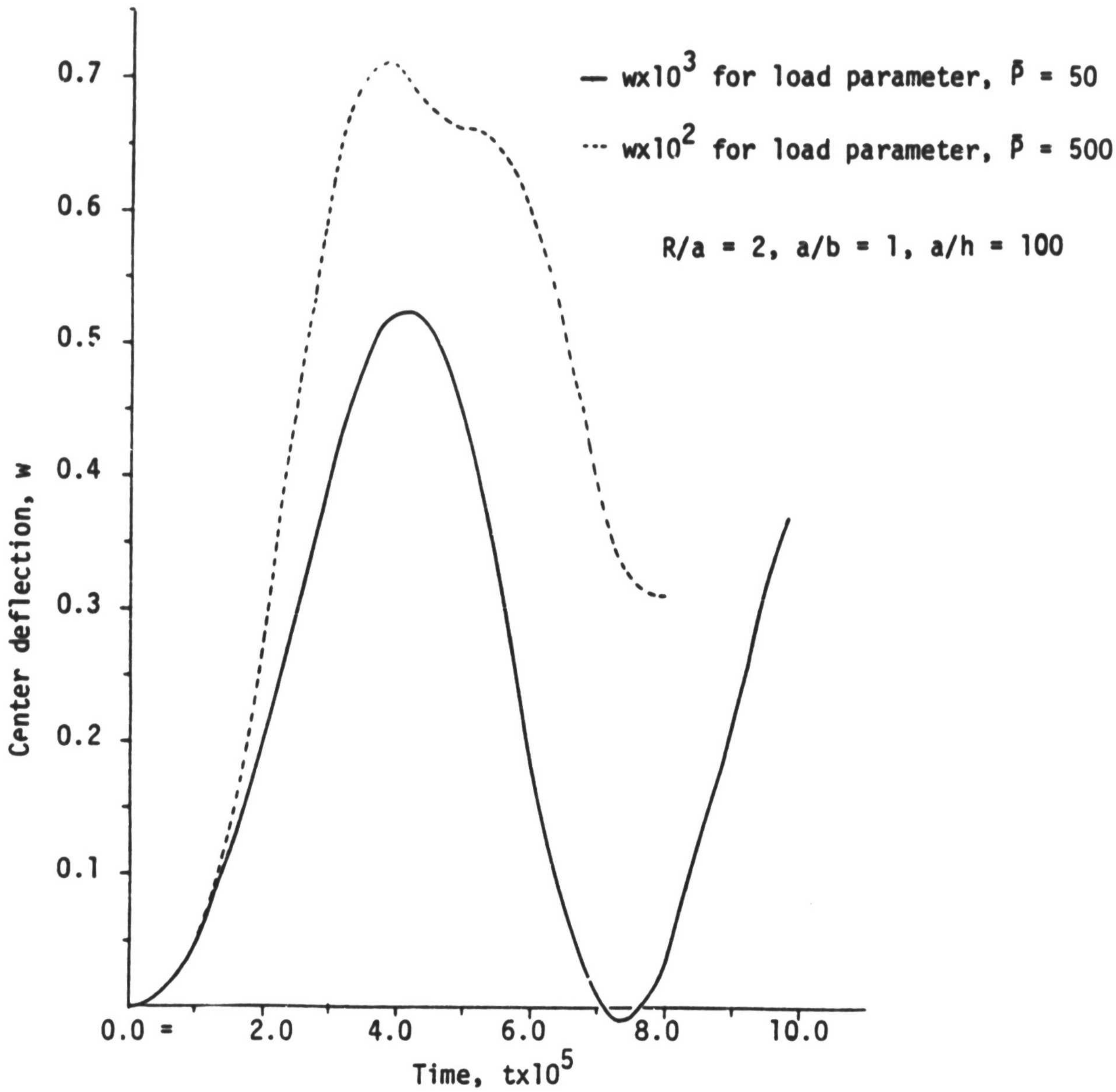

Figure 5.10 Center deflection versus time for two-layer angle-ply [45°/-45°] spherical shell under uniformly distributed step loading.

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary of the Present Study

A special three-dimensional element based on the total Lagrangian description of the motion of a layered anisotropic composite medium is developed, validated, and employed to analyze composite shells. The element has the following options:

- Geometrically linear and nonlinear analyses
- Static and transient analyses
- Natural vibration (linear) analyses
- Plate and shell elements
- Arbitrary loading and boundary conditions
- Arbitrary lamination schemes and lamina properties

The element can be used, with minor changes, in any existing general purpose program.

6.2 Conclusions

The present 3-D degenerated element has computational simplicity over a fully three-dimensional element, such as those developed in [49-50], and the element accounts for full geometric nonlinearities in contrast to 2-D elements based on shell theories. As demonstrated via numerical examples, the deflections obtained by the 2-D shell element deviate from those obtained by the 3-D element for deep shells. Further, the 3-D element can be used to model general shells that are not necessarily doubly-curved. For example, the vibration of twisted plates cannot be studied using the 2-D shell element discussed in Chapter 2. Of course, the 3-D degenerated element is computationally more demanding than the 2-D shell theory element for a given problem.

In summary, the present 3-D element is an efficient element for the analysis of layered composite plates and shells undergoing large displacements and transient motion.

6.3 Recommendations for Additional Study

The 3-D element presented herein can be modified to include thermal stress analysis capability and material nonlinearities. While the inclusion of thermal stresses is a simple exercise, the inclusion of nonlinear material effects is a difficult task. An acceptable material model could be a generalization of the Ramberg-Osgood relation to a layered anisotropic medium. Areas that require further study are the inclusion of damping effects, which can be more significant than the shear deformation effects, and material damage effects.

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REFERENCES

1. P. M. Naghdi, "A Survey of Recent Progrress in the Theory of Elastic Shells," Applied Mechanics Review Vol. 9, 1956, pp. 365.

2. C. W. Bert, "Analysis of Shells," Structural Design and Analysis, Part I, C. C. Chamis (ed.), Vol. 7 of Composite Materials, L. J. Broutman and R. H. Krock (eds.), Academic Press, New York, 1974.

3. S. A. Ambartsumyan, "Calculation of Laminated Anisotropic Shells," Izvestia Akademiia Nauk Armenskoi SSR, Ser. Fiz. Mat. Est. Tekh. Nauk., Vol. 6, No. 3, 1953, pp. 15.

4. S. A. Ambartsumyan, Theory of Anisotropic Shells, Moscow, 1961; English Translation, NASA TT F-118, May 1964.

5. S. B. Dong, K. S. Pister and R. L. Taylor, "On the Theory of Laminated Anisotropic Shells and Plates," Journal of Aerospace Sciences, Vol. 29, 1962, pp. 969-975.

6. Y. Stavsky, "Bending and Stretching of Laminated Anisotropic Plates," Journal of the Engineering Mechanics Division, Proc. ASCE, Vol. 87, No. EM6, 1961, pp. 31.

7. S. Cheng and B. P. C. Ho, "Stability of Heterogeneous Aelotropic Cylindrical Shells Under Combined Loading," American Institute of Aeronautics and Astronautics Journal, Vol. 1, No. 4, 1963, pp. 892-898.

8. O. E. Widera and S. W. Chung, "A Theory for Non-Homogeneous Anisotropic Cylindrical Shells," Journal of Applied Mathematics and Physics (ZAMP), Vol. 21, 1970, pp. 378-399.

9. S. T. Gulati and F. Essenburg, "Effects of Anisotropy in Axisymmetric Cylindrical Shells," Journal of Applied Mechanics, Vol. 34, Sept. 1967, pp. 659-666.

10. J. A. Zukas and J. R. Vinson, "Laminated Transversely Isotropic Cylindrical Shells," Journal of Applied Mechanics, Vol. 38, 1971, pp. 400-407.

11. J. M. Whitney and C. T. Sun, "A Refined Theory for Laminated Anisotropic, Cylindrical Shells," Journal of Applied Mechanics, Vol. 41, 1974, pp. 471-476.

12. G. E. O. Widera and D. L. Logan, "Refined Theories for Nonhomogeneous Anisotropic Cylindrical Shells: Part I-Derivation," Journal of the Engineering Mechanics Division Proc. ASCE, Vol. 106, No. EM6, 1980, pp. 1053-1074.

13. G. E. O. Widera and D. L. Logan, "Refined Theories for Nonhomogeneous Anisotropic Cylindrical Shells: Part II-Application," Journal of the Engineering Mechanics Division Proc. ASCE, Vol. 106, No. EM6, 1980, pp. 1075-1090.

14. S. B. Dong, "Analysis of Laminated Shells of Revolution," Journal of the Engineering Mechanics Division Proc. ASCE, Vol. 92, EM6, 1966, pp. 135.

15. S. B. Dong and L. G. Selna, "Natural Vibrations of Laminated Orthotropic Shells of Revolution," Journal of Composite Materials, Vol. 4, No. 1, 1970, pp. 2-19.

16. E. A. Wilson and B. Parsons, "The Finite Element Analysis of Filament-Reinforced Axisymmetric Bodies," Fibre Science and Technology, Vol. 2, 1969, pp. 155-156.

17. L. A. Schmit, Jr., and G. R. Monforton, "Finite Element Analysis of Sandwich Plate and Laminated Shells with Laminated Forces," American Institute of Aeronautics and Astronautics Journal, Vol. 8, 1970, pp. 1454-1461.

18. G. L. Thompson and C. W. Bert, "Finite Element Analysis for Free Vibration of General Anisotropic Laminated Thin Shells," Composite Materials in Engineering Design (Proc. 6th St. Louis Symposium, May 11-12, 1972), Norton, B. R. (ed.), American Society for Metals Park, Ohio, 1973, pp. 72-80.

19. S. C. Panda and R. Natarajan, "Finite Element Analysis of Laminated Shells of Revolution," Computers and Structures, Vol. 6, 1976, pp. 61-64.

20. K. N. Shivakumar and A. V. Krishna Murty, "A High Precision Ring Element for Vibrations of Laminated Shells," Journal of Sound and Vibration, Vol. 58, No. 3, 1978, pp. 311-318.

21. K. P. Rao, "A Rectangular Anisotropic Shallow Thin Shell Finite Element," Computer Methods in Applied Mechanics and Engineering, Vol. 15, 1978, pp. 13-83.

22. P. Seide and P. H. H. Chang, "Finite Element Analysis of Laminated Plates and Shells," NASA CR-157106, 1978, 132 pp.

23. A. Venkatesh and K. P. Rao, "A Doubly Curved Quadrilateral Finite Element for the Analysis of Laminated Anisotropic Thin Shell of Revolution," Computers and Structures, Vol. 12, 1980, pp. 825-832.

24. H. S. Hsu, J. N. Reddy and C. W. Bert, "Thermoelasticity of Circular Cylindrical Shells Laminated of Bimodulus Composite Materials," Journal of Thermal Stresses, Vol. 4, No. 2, 1981, pp. 115-127.

25. J. N. Reddy, "Bending of Laminated Anisotropic Shells by a Shear Deformable Finite Element," Fibre Science and Technology, Vol. 17, pp. 9-24, 1982.

26. G. Horrigmoe and P. G. Bergan, "Incremental Variational Principles and Finite Element Models for Nonlinear Problems," Computer Methods in Applied Mechanics and Engineering, Vol. 7, 1976, pp. 201-217.

27. W. Wunderlich, "Incremental Formulations for Geometrically Nonlinear Problems," Formulations and Algorithm in Finite Element Analysis, by K. J. Bathe, J. T. Oden and W. Wunderlich ed., pp. 193-239.

28. A. Stricklin, W. E. Haisler and W. A. Von Riesemann, "Evaluation of Solution Procedures for Material and/or Geometrically Nonlinear Structural Analysis," AIAA Journal, Vol. 11, 1973, pp. 292-299.

29. A. K. Noor and S. J. Hartley, "Nonlinear Shell Analysis Via Mixed Isoparametric Elements," Computers and Structures, Vol. 7, 1977, pp. 615-626.

30. T. Y. Chang and K. Sawamiphakdi, "Large Deformation Analysis of Laminated Shells by Finite Element Method," Computers and Structures, Vol. 13, 1981, pp. 331-340.

31. J. L. Sanders, Jr., "An Improved First-Approximation Theory for Thin Shells," NASA Technical Report R-24, June 1959.

32. C. H. Tsao, "Strains - Displacement Relations in Large Displacement Theory of Shells," AIAA Journal, Vol. 2, No. 11, 1964, pp. 2060-2062.

33. J. N. Reddy and W. C. Chao, "A Comparison of Closed-Form and Finite-Element Solutions of Thick Laminated Anisotropic Rectangular Plates," Nuclear Engineering and Design, Vol. 61, 1981, pp. 153-167.

34. J. N. Reddy, "On the Solutions to Forced Motions of Layered Composite Plates," AIAA Journal, Vol. 21, No. 4, 1983, pp. 621-629.

35. O. C. Zienkiewicz, R. L. Taylor and J. M. Too, "Reduced Integration Technique in General Analysis of Plates and Shells," International Journal for Numerical Methods in Engineering, Vol. 3, 1971, pp. 275-290.

36. B. Krakeland, "Nonlinear Analysis of Shells Using Degenerated Isoparametric Elements," Finite Elements in Nonlinear Mechanics, International Conference on Finite Elements in Nonlinear Solid and Structural Mechanics, Geilo, Norway, Aug. 1977, pp. 265-284.

37. K. J. Bathe, E. Ramm and E. L. Wilson, "Finite Element Formulations for Large Deformation Dynamic Analysis," International Journal for Numerical Methods in Engineering, Vol. 9, 1975, pp. 353-386.

38. G. A. Dupuris, H. D. Hibbit, S. F. McNamara and P. V. Marcal, "Nonlinear Material and Geometric Behavior of Shell Structures," Computers and Structures, Vol. 1, 1971, pp. 223-239.

39. G. Horrigmoe and P. G. Bergan, "Nonlinear Analysis of Free-Form Shells by Flat Finite Elements," Computer Methods in Applied Mechanics and Engineering, Vol. 16, 1978, pp. 11-35.

40. S. Ahmad, B. M. Irons and O. C. Zienkiewicz, "Analysis of Thick and Thin Shell Structures by Curved Finite Elements," International Journal for Numerical Methods in Engineering, 2, 1970, pp. 419-451.

41. R. E. Nickell,"On the Stability of Approximation Operators in Problems of Structural Dynamics," Int. J. Solids and Structures, Vol. 7, pp. 301-319, 1971.

42. R. E. N ickell, "Direct Integration Methods in Structural Dynamics," J. Engng. Mech. Div. ASCE, Vol. 99, No. EM2, 1972.

43. G. L. Goudreau, and R. L. Taylor, "Evaluation of Numerical Integration Methods in Elastodynamics," J. Computer Methods in Applied Mechanics and Engineering, Vol. 2, No. 1, pp. 69-97, 1973.

44. T. B. Belytschko and B. J. Hsieh, "Nonlinear Transient Finite Element Analysis with Convected Coordinates," International Journal for Numerical Methods in Engineering, Vol. 7, pp. 255-272, 1973.

45. T. J. R. Hughes, and W. K. Liu, "Implicit-Explicit Finite Elements in Transient Analysis: Stability Theory, and Implementation and Numerical Examples," Vol. 45, No. 2, pp. 371-378, 1978.

46. S. P. Chan, H. L. Cox and W. A. Benfield, "Transient Analysis of Forced Vibrations of Complex Structural Mechanical Systems," Journal of Royal Aeronant Soc., Vol. 66, pp. 457-460, 1962.

47. L. Fox and E. T. Goodwin, "Some New Methods for the Numerical Integration of Ordinary Differential Equations," Proc. Camb. Phil. Soc. Math. Phys. Sci. Vol. 45, 1949, pp. 373-388.

48. O. C. Zienkiewicz, The Finite Element Method, 3rd ed., McGraw-Hill Book Company.

49. G. S. Dhatt, "Instability of Thin Shells by the Finite Element Method," IASS Symposium for Folded Plates and Prismatic Structures, Vienna, 1970.

50. R. H. Leicester, "Large Elastic Deformations and Snap-Through of Shallow Doubly-Curved Shells," Ph.D. Thesis, Department of Civil Engineering, University of Illinois, Urbana, 1966.

51. J. A. Stricklin, J. E. Martinez, J. R. Tillerson, J. H. Hong, and W. E. Haisler, "Nonlinear Dynamic Analysis of Shells of Revolution by Matrix Displacement Method," AIAA Journal, Vol. 9, No. 4, April 1974, pp. 629-6[illegible]

52. S. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells, McGraw-Hill, New York, 1959.

53. J. N. Reddy, "Dynamic (Transient) Analysis of Layered Anisotropic Composite-Material Plates," International Journal for Numerical Methods in Engineering, Vol. 19, No. 2, pp. 237-255, 1983.

54. J. R. Dana and R. M. Barker, "Three-Dimensional Analysis for the Stress Distribution Near Circular Holes in Laminated Composites," Research Report No. VPI-E-74.18, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1974.

55. J. N. Reddy and T. Kuppusamy, "Analysis of Layered Composite Plates by Three-Dimensional Elasticity Theory," Research Report No. VPI-E-82.31, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1982.

APPENDIX I: ELEMENT STIFFNESS AND MASS MATRICES

$$[K^{11}] = A_{11}[S^{11}] + (A_{16} + c_oB_{16})([S^{12}] + [S^{12}]^T)$$

$$+ (A_{66} + 2c_oB_{66} + c_o^2D_{66})[S^{22}] + \frac{1}{R_1^2} A_{55}[S^{oo}]$$

$$[K^{12}] = (A_{16} - c_oB_{16})[S^{11}] + (A_{66} - c_o^2D_{66})[S^{12}]^T + A_{12}[S^{12}]$$

$$+ (A_{26} + c_oB_{26})[S^{22}] + \frac{1}{R_1R_2} A_{45} [S^{oo}]$$

$$[K^{13}] = (\frac{A_{11}}{R_1} + \frac{A_{12}}{R_2})[S^{10}] + [\frac{A_{16}}{R_1} + \frac{A_{26}}{R_2} + c_o(\frac{B_{16}}{R_1} + \frac{B_{26}}{R_2})] [S^{20}]$$

$$- \frac{1}{R_1} (A_{45}[S^{02}] + A_{55}[S^{01}])$$

$$[K^{14}] = B_{11}[S^{11}] + B_{16}[S^{12}] + (B_{16} + c_oD_{16})[S^{12}]^T$$

$$+ (B_{66} + c_oD_{66})[S^{22}] - \frac{A_{55}}{R_1} [S^{oo}]$$

$$[K^{15}] = B_{12}[S^{12}] + B_{16}[S^{11}] + (B_{26} + c_oD_{26})[S^{22}]$$

$$+ (B_{66} + c_oD_{66})[S^{12}]^T - \frac{1}{R_1} A_{45} [S^{oo}]$$

$$[K^{22}] = A_{22}[S^{22}] + (A_{26} - c_oB_{26})([S^{12}] + [S^{12}]^T)$$

$$+ (A_{66} - 2c_oB_{66} + c_o^2D_{66})[S^{11}] + \frac{A_{44}}{R_2^2} [S^{oo}]$$

$$[K^{23}] = [\frac{A_{16}}{R_1} + \frac{A_{26}}{R_2} - c_o (\frac{B_{16}}{R_1} + \frac{B_{26}}{R_2})][S^{10}]$$

$$+ (\frac{A_{12}}{R_1} + \frac{A_{22}}{R_2})[S^{20}] - \frac{1}{R_2} (A_{44}[S^{02}] + A_{45}[S^{01}])$$

$$[K^{24}] = (B_{16} - c_oD_{16})[S^{11}] + (B_{66} - c_oD_{66})[S^{12}] + B_{12}[S^{12}]$$

$$+ B_{26} [S^{22}] - \frac{1}{R_2} A_{45} [S^{oo}]$$

$$[K^{25}] = B_{22}[S^{22}] + B_{26}[S^{12}]^T + (B_{26} - c_oD_{26})[S^{12}]$$

$$+ (B_{66} - c_oD_{66})[S^{11}] - \frac{A_{44}}{R_2} [S^{oo}]$$

$$[K^{52}] = B_{22}[S^{22}] + B_{26}[S^{12}] + (B_{26} - c_o D_{26})[S^{12}]^T + (B_{66} - c_o D_{66})[S^{11}] - \frac{A_{44}}{R_2}[S^{00}]$$

$$[K^{33}] = A_{45}[S^{12}] + A_{55}[S^{11}] + A_{44}[S^{22}] + A_{45}[S^{12}]^T + [(\frac{A_{11}}{R_1} + \frac{A_{12}}{R_2})\frac{1}{R_1} + (\frac{A_{12}}{R_1} + \frac{A_{22}}{R_2})\frac{1}{R_2}][S^{00}]$$

$$[K^{34}] = A_{55}[S^{10}] + A_{45}[S^{20}] + (\frac{B_{11}}{R_1} + \frac{B_{12}}{R_2})[S^{01}] + (\frac{B_{16}}{R_1} + \frac{B_{26}}{R_2})[S^{02}]$$

$$[K^{35}] = A_{45}[S^{10}] + A_{44}[S^{20}] + (\frac{B_{12}}{R_1} + \frac{B_{22}}{R_2})[S^{02}] + (\frac{B_{16}}{R_1} + \frac{B_{26}}{R_2})[S^{01}]$$

$$[K^{44}] = D_{11}[S^{11}] + D_{16}([S^{12}] + [S^{12}]^T) + D_{66}[S^{22}] + A_{55}[S^{00}]$$

$$[K^{45}] = D_{12}[S^{12}] + D_{16}[S^{11}] + D_{26}[S^{22}] + D_{66}[S^{12}]^T + A_{45}[S^{00}]$$

$$[K^{55}] = D_{26}[S^{12}] + D_{66}[S^{11}] + D_{22}[S^{22}] + D_{26}[S^{12}]^T + A_{44}[S^{00}]$$

$$[M^{11}] = [M^{22}] = [M^{33}] = P_1[S^{00}]$$

$$[M^{44}] = [M^{55}] = P_3[S^{00}]$$

$$[M^{14}] = [M^{15}] = \bar{P}[S^{00}], \quad \bar{P} = P_3[1/R_1 + 1/R_2]$$

$$S_{ij}^{\alpha\beta} = \int_{\Omega^e} D_\alpha \psi_i D_\beta \psi_j dx_1 dx_2 , \ (\alpha,\beta = 0,1,2) , \ D_\alpha \equiv \frac{\partial}{\partial x_\alpha} , \ D_o = 1.$$

APPENDIX II: NOMENCLATURE

A	- area of element
A_{ij}	- stretching stiffness matrix
α, β	- dimensionless parameters of generalized acceleration
α_i	- surface metric
ϵ_i	- strain in the i-th direction
ϵ_i^o	- strain in the ith direction at the reference plane
B_{ij}	- bending-stretching stiffness matrix
C_t	- configuration at time t
$C_{ijk\ell}$	- elasticity tensor
D	- $Eh^3/12(1 - \nu^2)$
D_{ij}	- bending stiffness matrix
$\{\Delta\}$	- generalized displacement vector
c_o	- $\frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
E_i	- Young's modulus
$\hat{\underset{\sim}{E}}_T$	- unit vector along the global axes
$\hat{\underset{\sim}{e}}$	- unit vector along the local axes
e_{ij}	- linear incremental strain
ξ_1, ξ_2, ξ	- curvilinear coordinate system
$\{F\}$	- force vector
ϕ_i	- rotations of the reference surface with respect to ξ_2-axis
h	- thickness
$[K]$	- stiffness matrix
K_i	- shear correction factor
L	- number of layer
M_i	- moment resultatnt
$[M]$	- mass matrix

ν_{ij}	- Poisson's ratio
N_i	- stress resultant
η_{ij}	- nonlinear incremental strain
P_i	- inertias
q_i	- distributed load
Q_i	- shear force resultant
θ_1, θ_2	- rotations about unit vector $\hat{\underset{\sim}{e}}_1, \hat{\underset{\sim}{e}}_2$
$\{R\}$	- balance force vector
R_1, R_2	- radii of curvature
ρ	- density
S_{ij}	- 2nd Piola Kirchhoff stress tensor
$[S]$	- stress matrix
$\{\hat{S}_j\}$	- stress vector
dS	- length of line element of the shell
σ_i	- stress vector
ψ_i	- interpolation function at node i
$[T]$	- transformation matrix between displacement vector $\{u\}$ and generalized displacement vector $\{\Delta\}$
T_i	- traction component
Δt	- time increment
$\underset{\sim}{u}$	- incremental displacement vector
${}^t\underset{\sim}{u}$	- displacement vector at time t
$\underset{\sim}{u}_{,\alpha}$	- displacement gradient
$\underset{\sim}{v}_i$	- vector along the local axes
V	- volume
${}^t\underset{\sim}{x}$	- coordinate at time t
δW	- virtual work due to external loads
$\{\Delta\}$	- column of generalized nodal displacements

```
C                                                                      DGN00010
C                       P R O G R A M   SHEL3D                         DGN00020
C......................................................................DGN00030
C                                                                      DGN00040
C       THIS PROGRAM IS DEVELPED TO ANALYZE LAMINATED ANISOTRPIC       DGN00050
C       STRUCTURES (PLATES AND SHELLS) WITH GEOMETRIC NONLINEARITY.    DGN00060
C       THE TOTAL LAGRANGIAN DESCRIPTION WITH INCREMENTAL VARIATIONAL  DGN00070
C       IS USED.  THE THREE-DIMENSIONAL DEGENERATED ELEMENT IS USED    DGN00080
C       TO MODEL LAMINATED PLATES AND SHELLS.                          DGN00090
C                                                                      DGN00100
C......................................................................DGN00110
C                                                                      DGN00120
C       D E S C R I P T I O N O F   V A R I A B L E S                  DGN00130
C                                                                      DGN00140
C       COOD(I,J)......GLOBAL COORDINATE OF NODE I IN J-TH DIRECTION.  DGN00150
C       WO,W1,W2 ......VECTOR OF DISPLACEMENT, VELOCITY AND            DGN00160
C                      ACCELERATION OF EACH ELEMENT.                   DGN00170
C       GEO,GE1,GE2....VECTOR OF DISPLACEMENT, VELOCITY AND            DGN00180
C                      ACCELERATION OF THE WHOLE STRUCTURE.            DGN00190
C       DTH............VECTOR INDICATES VARIOUS TIME STEP.             DGN00200
C       NTIME.......... NUMBER OF TIME STEP TO BE PROCESSED.           DGN00210
C       NTMAX..........MAXIMUM NUMBER OF ITERATION USED IN NONLINEAR   DGN00220
C                      ANALYSIS.                                       DGN00230
C       THETA(I).......ORIENTATION OF I-TH LAYER.                      DGN00240
C       NGP,LGP........NO. OF INTEGRATION POINT FOR BENDING PARTS AND DGN00250
C                      SHEAR PARTS.                                    DGN00260
C       IPRINT ........1 : PRINT THE ELEMENT STIFFNESS MATRIX.         DGN00270
C                      0 : NOT PRINT ELEMENT STIFFNESS MATRIX.         DGN00280
C       NLS............NO. OF LOAD STEP.                               DGN00290
C       LTYPE..........FLAG TO INDICATE SHELL TYPE                     DGN00300
C                      0 : FLAT PLATE                                  DGN00310
C                      1 : CYLINDRICAL SHELL                           DGN00320
C                      2 : SPHERICAL SHELL                             DGN00330
C       IPROB..........0 : STATIC PROBLEM.                             DGN00340
C                      1 : DYNAMIC PROBLEM.                            DGN00350
C       TH1,TH2........ANGLE OF SPAN IN RADIAN OF X AND Y DIRECTION.   DGN00360
C       RD1,RD2........RADIUS MEASURED IN PRINCIPAL X AND Y DIRECTION.DGN00370
C       ALFA, BETA AND DELTA............PARAMETERS USED IN DYNAMIC     DGN00380
C                      ANALYSIS.                                       DGN00390
C       ELXYZ(I,J).....GLOBAL COORDINATE IN J-TH DIRECTION OF I NODE.  DGN00400
C       V1,V2,V3.......LOCAL COORDINATE VECTORS AFTER EACH TIME STEP.  DGN00410
C       V1A,V2B,V3C....LOCAL COORDINATE VECTORS AT CURRENT TIME.       DGN00420
C       VA,VB,VC.......LOCAL COORDINATE VECTORS AT TIME 0.             DGN00430
C       VEL,VAC........VELOCITY AND ACCELERATION OF V3 AT TIME (N+1)*TDGN00440
C       VELO,VACO......VELOCITY AND ACCELERATION OF V3 AT TIME N*T.    DGN00450
C       NNM,NEM........NO. OF NODE AND ELEMENT IN THIS MESH.           DGN00460
C       NOD............BOOLEAN MATRIX.                                 DGN00470
C       RHO............DENSITY.                                        DGN00480
C                                                                      DGN00490
C                                                                      DGN00500
C......................................................................DGN00510
C                                                                      DGN00520
      IMPLICIT REAL*8(A-H,O-Z)                                         DGN00530
      COMMON/MATL/A(2,3,6,6),CM(9,6,6),CS(9,6,6)                       DGN00540
      COMMON/MESH3/COOD(50,3),ELXYZ(9,3),NOD(9,9)                      DGN00550
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      COMMON/ICV/V1(50,3),V2(50,3),V3(50,3),VA(50,3),VB(50,3),VC(50,3)  DGN00560
     1           ,V1A(50,3),V2B(50,3),V3C(50,3)                          DGN00570
      COMMON/STIF1/STIF(45,45),ELP(45),W(45),ELQ(45),W0(45),W1(45)      DGN00580
     1           ,W2(45),VAC(50,3),VEL(50,3),VAC0(50,3),VEL0(50,3)      DGN00590
      DIMENSION THETA(10),GF(145),GSTIF(145,145),GINSF(145,145),Z(10),  DGN00600
     1           GW(145),GP(145),GU(145),IBDY(90),VBDY(90),GB(145),GE0( DGN00610
     2           145),GE1(145),GE2(145),DIM(3),GP1(145),GC(145)         DGN00620
      READ(5,610) DIM(1),DIM(2),DIM(3)                                  DGN00630
      READ(5,620) NDF,NPE,NEM,NNM,NBDY                                  DGN00640
      READ(5,610)THETA(1),THETA(2),THETA(3),THETA(4)                    DGN00650
      READ(5,610)TH1,TH2,RD,RD1,RD2                                     DGN00660
      READ(5,610)E1,E2,G12,G13,G23,ANU12                                DGN00670
      READ(5,620)IEL,NGP,LGP,NES1,NES2,ITYPE                            DGN00680
      READ(5,620)NTIME,NTMAX,IPRINT,MES,NKMAX,ILOAD,LAYER               DGN00690
      READ(5,610)RHO                                                    DGN00700
      READ(5,610)EPS,XL,YL,T,PR                                         DGN00710
C     DO 2 I=1,NNM                                                      DGN00720
C   2 READ (5,10)(COOD(I,J),J=1,3)                                      DGN00730
C     DO 3 I=1,NNM                                                      DGN00740
C   3 READ (5,10)(V3(I,J),J=1,3)                                        DGN00750
      CALL MESHEL(TH1,TH2,NES1,NES2,RD,ITYPE,XL,YL)                     DGN00760
      DO 4 I=1,NEM                                                      DGN00770
    4 READ (5,20)(NOD(I,J),J=1,NPE)                                     DGN00780
      WRITE(6,560)NBDY                                                  DGN00790
      READ (5,22)(IBDY(I),I=1,NBDY)                                     DGN00800
      WRITE(6,23)(IBDY(I),I=1,NBDY)                                     DGN00810
   23 FORMAT(/////,2X,'B O U N D A R Y   C O N D I T I O N ',/,2X,20    DGN00820
     C     I5,/,2X,20I5,/,2X,20I5)                                      DGN00830
C.......................................................................DGN00840
C     NA2..... FLAG TO SPECIFY ISOTROPIC OR NOT, IT CAN REDUCE         .DGN00850
C              INTEGRATION TIME IN STIFF SUBROUTINE.                   .DGN00860
C.......................................................................DGN00870
      NA2=1                                                             DGN00880
      IF(E1.EQ.E2) NA2=0                                                DGN00890
      ALFA=0.5                                                          DGN00900
      BETA=0.25*(0.5+ALFA)**2                                           DGN00910
      GAMA=-2.0/3.0                                                     DGN00920
      NEQ=NDF*NNM                                                       DGN00930
      NN=NDF*NPE                                                        DGN00940
C     P0=(-1.0)*(E2*T**4/(XL**4))*50.0*PR                               DGN00950
      P0=(-1.0)*(E2*T**4/((2.0*XL)**4))*50.0*PR                         DGN00960
C     P0=-2.85                                                          DGN00970
C     P0=0.5D-2                                                         DGN00980
C     P0=-1.0D2                                                         DGN00990
C     P0=-100.0                                                         DGN01000
  157 CONTINUE                                                          DGN01010
C.......................................................................DGN01020
C     CA1, CA2 ARE THE NORMAL AND ROTARY INERTIA COEFFICIENTS.         .DGN01030
C.......................................................................DGN01040
      CA1=RHO*2.0                                                       DGN01050
      CA3=RHO*2.0**3/12.0                                               DGN01060
      DP=P0                                                             DGN01070
      OP=P0                                                             DGN01080
      WRITE(6,580)ILOAD                                                 DGN01090
  580 FORMAT(//,2X,'L O A D I N G   T Y P E (1: UNIFORM, 2: SINUSOIDAL, DGN01100
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     10: CONCENTRATED)',/,2X,'L L O A D - ',I3)                        DGN01110
      WRITE(6,1)NDF,NPE,NEM,NNM,XL,YL,RD1,RD2                           DGN01120
      WRITE(6,570)NGP,LGP                                               DGN01130
      WRITE(6,6)E1,E2,G12,G13,G23,ANU12                                 DGN01140
      WRITE(6,11)LAYER,(THETA(I),I=1,LAYER)                             DGN01150
      WRITE(6,12)I,EPS                                                  DGN01160
      WRITE(6,16)((COOD(I,J),J=1,3),I=1,NNM)                            DGN01170
      WRITE(6,21)((NOD(I,J),J=1,NPE),I=1,NEM)                           DGN01180
C...............................................................DGN01190
C         FIND OUT THE LOCAL COORDINATES FOR EACH NODE.                 DGN01200
C...............................................................DGN01210
      DO 30 I=1,NNM                                                     DGN01220
      DO 33 K=1,3                                                       DGN01230
      VAC(I,K)=0.0                                                      DGN01240
      VEE(I,K)=0.0                                                      DGN01250
      VACO(I,K)=0.0                                                     DGN01260
      VEEO(I,K)=0.0                                                     DGN01270
   33 CONTINUE                                                          DGN01280
      V1(I,1)=V3(I,3)                                                   DGN01290
      V1(I,2)=0.0                                                       DGN01300
      V1(I,3)=-V3(I,1)                                                  DGN01310
      V2(I,1)=-V3(I,2)*V3(I,1)                                          DGN01320
      V2(I,2)=V3(I,1)**2+V3(I,3)**2                                     DGN01330
      V2(I,3)=-V3(I,2)*V3(I,3)                                          DGN01340
      SUM1=DSQRT(V1(I,1)**2+V1(I,2)**2+V1(I,3)**2)                      DGN01350
      SUM2=DSQRT(V2(I,1)**2+V2(I,2)**2+V2(I,3)**2)                      DGN01360
      SUM3=DSQRT(V3(I,1)**2+V3(I,2)**2+V3(I,3)**2)                      DGN01370
      DO 30 J=1,3                                                       DGN01380
      V1(I,J)=V1(I,J)/SUM1                                              DGN01390
      V2(I,J)=V2(I,J)/SUM2                                              DGN01400
      V3(I,J)=V3(I,J)/SUM3                                              DGN01410
   30 CONTINUE                                                          DGN01420
      DO 62 I=1,NNM                                                     DGN01430
      DO 62 J=1,3                                                       DGN01440
      V1A(I,J)=V1(I,J)                                                  DGN01450
      V2B(I,J)=V2(I,J)                                                  DGN01460
      V3C(I,J)=V3(I,J)                                                  DGN01470
      VA(I,J)=V1(I,J)                                                   DGN01480
      VB(I,J)=V2(I,J)                                                   DGN01490
      VC(I,J)=V3(I,J)                                                   DGN01500
   62 CONTINUE                                                          DGN01510
      WRITE(6,136)                                                      DGN01520
      DO 126 II=1,NNM                                                   DGN01530
  126 WRITE(6,10)(V1(II,JI),JI=1,3)                                     DGN01540
      WRITE(6,137)                                                      DGN01550
      DO 127 II=1,NNM                                                   DGN01560
  127 WRITE(6,10)(V2(II,JI),JI=1,3)                                     DGN01570
      WRITE(6,138)                                                      DGN01580
      DO 128 II=1,NNM                                                   DGN01590
  128 WRITE(6,10)(V3(II,JI),JI=1,3)                                     DGN01600
  136 FORMAT(//,2X,'V E C T O R  I N  1 - D I R E C T I O N ',/)       DGN01610
  137 FORMAT(//,2X,'V E C T O R  I N  2 - D I R E C T I O N ',/)       DGN01620
  138 FORMAT(//,2X,'V E C T O R  I N  3 - D I R E C T I O N ',/)       DGN01630
      DO 250 I=1,NEQ                                                    DGN01640
  250 GF(I)=0.0                                                         DGN01650
```

```
C.......................................................................
C     ITR IS A FLAG TO INDICATE THE NUMBER OF TIME TO FIND
C     THE TANGENTIAL STIFFNESS AT EACH LOADING STEP
C.......................................................................
      ITR=1
C.......................................................................
C     CACULATE THE CONSTITUTIVE RELATIONSHIP.
C.......................................................................
      CALL MATPRP(E1,E2,ANU12,G12,G13,G23,L,LAYER,NNM,THETA,Z,NGP,LGP)
      DO 300 NP=1,NLS
      WRITE(6,550)PO
C.......................................................................
C     INITIALIZATION OF THE MATRIXS AND VECTORS.
C.......................................................................
      DO 251 I=1,NEQ
      GLO(I)=0.0
      GL1(I)=0.0
      GL2(I)=0.0
      DO 251 J=1,NEQ
      GINST(I,J)=0.0
      GSTIF(I,J)=0.0
  251 CONTINUE
C.......................................................................
C     CALCULATION OF THE PARAMETER USED IN DYNAMIC PROBLEMS.
C.......................................................................
      ET=DTM(NP)
      ET2=ET*ET
      A0=1.0/BETA/ET2
      A1=1.0/BETA/ET
      A2=1.0/2.0/BETA-1.
      A3=A0
      A4=-A1
      A5=-A2
      A6=ET*(1.0-ALFA)
      A7=ALFA*ET
      TIME=0.0
      WRITE(6,323)ALFA,BETA,GAMA,A0,A1,A2,A3,A4,A5,A6,A7,RHO
  323 FORMAT(//,2X,'A L F A , B E T A , G A M A =',3E13.5,/,2X,'A 0, A
     11, A 2,....   ',8E13.5,/,2X,'D E N S I T Y = ',E13.5,/)
      WRITE(6,313)
      DO 290 NIT=1,NTIME
      TIME=TIME+ET
      ITER=0
C.......................................................................
C     IFLAG INDICATES TO UPDATE THE TANGENTIAL STIFFNESS MATRIX
C              IFLAG = 0 UPDATE THE TANGENTIAL STIFFNESS MATRIX.
C              IFLAG = 1 DON'T UPDATE THE TANGENTIAL STIFFNESS MATRIX.
C.......................................................................
      NSS=NIT
      IF(IPROB.EQ.0)NSS=NP
      IFLAG=0
      NIA=NSS/10
      NIB=NIA*10
      IF(NIB.NE.NSS) IFLAG=1
      IF(NSS.EQ.1) IFLAG=0
```

```
  240 CONTINUE
      ITER=ITER+1
      IF(ITER.GT.NTMAX) GO TO 410
C.......................................................................
C     INITIALIZATIN OF THE GLOBAL MATRIXS.
C.......................................................................
      DO 40 I=1,NEQ
      GU(I)=0.0
      GB(I)=0.0
      GC(I)=GE(I)
      IF(IFLAG.NE.0)GO TO 40
      DO 45 J=1,NEQ
      GINST(I,J)=0.0
      GSTIF(I,J)=0.0
   45 CONTINUE
   40 CONTINUE
C.......................................................................
C     CACULATE THE GLOBAL MATRIX FOR EACH ELEMENT AND ASSEMBLE THEM.
C.......................................................................
      DO 50 N=1,NEM
      I=0
      DO 60 L=1,NPE
      NI=NOD(N,L)
      ELXYZ(L,1)=COOD(NI,1)
      ELXYZ(L,2)=COOD(NI,2)
      ELXYZ(L,3)=COOD(NI,3)
      LI=(NI-1)*NDF
      DO 60 J=1,NDF
      LI=LI+1
      I=I+1
      IF(PROB.EQ.0)GO TO 60
      W0(I)=GE0(LI)
      W1(I)=GE1(LI)
      W2(I)=GE2(LI)
   60 W(I)=GE(LI)
      CALL STIFF(NN,NPE,NDF,N,I,PO,ITER,ILOAD,NP,IFLAG,NGP,LGP,CA1,CA3,
     1      A0,A1,A2,NTT,NTMAX,NA2,IPROB)
      IF(IPRINT.EQ.1) GO TO 77
      IF(N.NE.1) GO TO 77
      PRINT 79,(ELP(II),II=1,NN)
      DO 78 II=20,25
   78 WRITE(6,79)(STIF(II,JJ),JJ=20,25)
   79 FORMAT(7(2X,E13.5))
   77 DO 70 I=1,NPE
      NR=(NOD(N,I)-1)*NDF
      DO 70 II=1,NDF
      NR=NR+1
      L=(I-1)*NDF+II
      GU(NR)=GU(NR)+ELP(L)
      IF(ILOAD.EQ.0) GO TO 489
      GB(NR)=GB(NR)+ELQ(L)
  489 IF(IFLAG.NE.0)GO TO 70
      DO 490 J=1,NPE
      NC=(NOD(N,J)-1)*NDF
      DO 490 JJ=1,NDF
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      M=(J-1)*NDF+JJ                                                    DGN02760
      NC=NC+1                                                           DGN02770
      GSTIF(NR,NC)=GSTIF(NR,NC)+STIF(L,M)                               DGN02780
  490 CONTINUE                                                          DGN02790
   70 CONTINUE                                                          DGN02800
   50 CONTINUE                                                          DGN02810
      IF(ILOAD.EQ.0) GB(63)=PO                                          DGN02820
C...................................................................... DGN02830
C     APPLY THE BOUNDARY CONDITIONS                                     DGN02840
C...................................................................... DGN02850
      CALL BOU(NRMAX,GSTIF,GU,GB,NEQ,NEQW,NBDY,IBDY,ITER,IFLAG)         DGN02860
C...................................................................... DGN02870
C     SOLVE THE EQUATION WITH THE NEWTON-RAPHSON METHOD.                DGN02880
C...................................................................... DGN02890
      IF(IFLAG.NE.0)GO TO 480                                           DGN02900
      CALL LINV1F(GSTIF,NEQW,NRMAX,GINSF,0,GW,IER)                      DGN02910
  480 DO 80 II=1,NEQW                                                   DGN02920
      GP(II)=0.0                                                        DGN02930
      DO 80 JI=1,NEQW                                                   DGN02940
      GP(II)=GP(II)+GINSF(II,JI)*(GU(JI)+GB(JI))                        DGN02950
   80 CONTINUE                                                          DGN02960
C...................................................................... DGN02970
C     PUT THE SPECIFIED BOUNDARY TERMS BACK TO THE GP VECTOR.           DGN02980
C...................................................................... DGN02990
      NBDW=NEQ-NEQW                                                     DGN03000
      K=0                                                               DGN03010
      DO 90 I=1,NEQ                                                     DGN03020
      DO 100 J=1,NBDW                                                   DGN03030
      L=IBDY(NBDY+1-J)                                                  DGN03040
      IF(L.EQ.I) GO TO 110                                              DGN03050
      IF(L.GT.I)GO TO 120                                               DGN03060
  100 CONTINUE                                                          DGN03070
  120 GF(I)=GF(I)+GP(I-K)                                               DGN03080
      GO TO 90                                                          DGN03090
  110 K=K+1                                                             DGN03100
      GF(I)=0.0                                                         DGN03110
   90 CONTINUE                                                          DGN03120
C...................................................................... DGN03130
C     CALCULATE THE NORM OF THE DIFFERENCE OF THE CONSECUTIVE SOLUTIONS.DGN03140
C...................................................................... DGN03150
  310 ERR=0.0                                                           DGN03160
      DO 130 II=3,NEQ,NDF                                               DGN03170
  130 ERR=ERR+(GF(II)-GC(II))**2                                        DGN03180
      ERR=DABS(DSQRT(ERR)/GF(63))                                       DGN03190
      IFLAG=1                                                           DGN03200
      IF(MIMAX.EQ.1) GO TO 321                                          DGN03210
C...................................................................... DGN03220
C     FIND OUT THE LOCAL COORDINATE VECTORS AT CURRENT TIME.            DGN03230
C...................................................................... DGN03240
346   N1=NDF-1                                                          DGN03250
      N2=NDF-2                                                          DGN03260
      DO 255 I=1,NNH                                                    DGN03270
      L=NDF*(I-1)+1                                                     DGN03280
      DO 170 J=1,3                                                      DGN03290
      V3C(I,J)=V3(I,J)+V1(I,J)*GF(L+N2)-V2(I,J)*GF(L+N1)                DGN03300
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  170 CONTINUE                                                    DGN03310
      V1A(I,1)=V3C(I,3)                                           DGN03320
      V1A(I,2)=0.0                                                DGN03330
      V1A(I,3)=-V3C(I,1)                                          DGN03340
      V2B(I,1)=-V3C(I,2)*V3C(I,1)                                 DGN03350
      V2B(I,2)=V3C(I,1)**2+V3C(I,3)**2                            DGN03360
      V2B(I,3)=-V3C(I,2)*V3C(I,3)                                 DGN03370
      SUM1=DSQRT(V1A(I,1)**2+V1A(I,2)**2+V1A(I,3)**2)             DGN03380
      SUM2=DSQRT(V2B(I,1)**2+V2B(I,2)**2+V2B(I,3)**2)             DGN03390
      SUM3=DSQRT(V3C(I,1)**2+V3C(I,2)**2+V3C(I,3)**2)             DGN03400
      DO 156 K=1,3                                                DGN03410
      V1A(I,K)=V1A(I,K)/SUM1                                      DGN03420
      V2B(I,K)=V2B(I,K)/SUM2                                      DGN03430
      V3C(I,K)=V3C(I,K)/SUM3                                      DGN03440
  156 CONTINUE                                                    DGN03450
  255 CONTINUE                                                    DGN03460
  241 IF(ERR.GT.EPS) GO TO 240                                    DGN03470
      IF(IPROB.EQ.0) GO TO 347                                    DGN03480
C.................................................................DGN03490
C     CALCULATE THE ANGLE VELOCITY AND ACCELERATION OF NORMAL VECTOR  .DGN03500
C     OF V3 AT EACH NODE.                                         .DGN03510
C.................................................................DGN03520
      DO 433 I=1,NNM                                              DGN03530
      NDF*(I-1)+1                                                 DGN03540
      433 J=1,3                                                   DGN03550
      (I,J)=A3*(GE(I+N2)*V1(I,J)-V2(I,J)*GE(I+N1))+A4*VELO(I,J)   DGN03560
     1          +A5*VACO(I,J)                                     DGN03570
      VEL(I,J)=VELO(I,J)+A6*VACO(I,J)+A7*VAC(I,J)                 DGN03580
      VACO(I,J)=VAC(I,J)                                          DGN03590
      VELO(I,J)=VEL(I,J)                                          DGN03600
  433 CONTINUE                                                    DGN03610
C.................................................................DGN03620
C     FIND OUT THE LOCAL COORDINATE VECTOR AT THE END OF TIME STEP    .DGN03630
C.................................................................DGN03640
  347 N1=NDF-1                                                    DGN03650
      N2=NDF-2                                                    DGN03660
      DO 55 I=1,NNM                                               DGN03670
      L=NDF*(I-1)+1                                               DGN03680
      DO 150 J=1,3                                                DGN03690
      V3(I,J)=V3(I,J)+V1(I,J)*GE(L+N2)-V2(I,J)*GE(L+N1)           DGN03700
  150 CONTINUE                                                    DGN03710
      V1(I,1)=V3(I,3)                                             DGN03720
      V1(I,2)=0.0                                                 DGN03730
      V1(I,3)=-V3(I,1)                                            DGN03740
      V2(I,1)=-V3(I,2)*V3(I,1)                                    DGN03750
      V2(I,2)=V3(I,1)**2+V3(I,3)**2                               DGN03760
      V2(I,3)=-V3(I,2)*V3(I,3)                                    DGN03770
      SUM1=DSQRT(V1(I,1)**2+V1(I,2)**2+V1(I,3)**2)                DGN03780
      SUM2=DSQRT(V2(I,1)**2+V2(I,2)**2+V2(I,3)**2)                DGN03790
      SUM3=DSQRT(V3(I,1)**2+V3(I,2)**2+V3(I,3)**2)                DGN03800
      DO 56 K=1,3                                                 DGN03810
      V1(I,K)=V1(I,K)/SUM1                                        DGN03820
      V2(I,K)=V2(I,K)/SUM2                                        DGN03830
      V3(I,K)=V3(I,K)/SUM3                                        DGN03840
   56 CONTINUE                                                    DGN03850
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   55 CONTINUE
  321 DO 430 I=1,NEQ
      IA=I+1
      IC=IA/5
      ID=I/5
      MC=IC*5
      MD=ID*5
      IF(IPROB.EQ.0) GO TO 432
      IF(IA.EQ.MC.OR.I.EQ.MD) GO TO 432
      GE0(I)=A3*(GE(I)-GE0(I))+A4*GE1(I)+A5*GE2(I)
      GE1(I)=GE1(I)+A6*GE2(I)+A7*GE0(I)
      GE2(I)=GE0(I)
  432 IF(IA.EQ.MC.OR.I.EQ.MD)GE(I)=0.0
      IF(IPROB.EQ.0) GO TO 430
      GE0(I)=GE(I)
  430 CONTINUE
  326 IF(IPRB.EQ.1)WRITE(6,311)TIME,GE(63),GE1(63),GE2(63),ERR,ITER
      IF(IPRB.EQ.1) GO TO 345
      WRITE(6,320)
  311 WRITE(6,330)
      WRITE(6,340)(GE(I),I=1,NEQ,NDF)
      WRITE(6,350)
      WRITE(6,340)(GE(I),I=2,NEQ,NDF)
      WRITE(6,360)
      WRITE(6,340)(GE(I),I=3,NEQ,NDF)
      WRITE(6,370)
      WRITE(6,340)(GE(I),I=4,NEQ,NDF)
      WRITE(6,380)
      WRITE(6,340)(GE(I),I=5,NEQ,NDF)
      WRITE(6,590)
  345 DO 245 N=1,1
      WRITE(6,600)N
      L=0
      DO 215 I=1,NPE
      NI=NOD(N,I)
      ELXYZ(I,1)=COOD(NI,1)
      ELXYZ(I,2)=COOD(NI,2)
      ELXYZ(I,3)=COOD(NI,3)
      LI=(NI-1)*NDF
      DO 215 J=1,NDF
      LI=LI+1
      L=L+1
  215 V(L)=GE(LI)
  245 CALL STRESS(NGP,N,L,NN,Z,NPE,LAYER)
  290 CONTINUE
      PO=PO+DP
  300 CONTINUE
      STOP
  410 WRITE(6,450)
  313 FORMAT(//,6X,'TIME',9X,'DEFL',9X,'VELTY',8X,'ACCL',9X,'EROR',9X
     1,'ITER'/)
  311 FORMAT(2X,5E13.5,I5)
  450 FORMAT(' ',5X,'********** C O N V E R G E N C E   I S   N O T   A C
     1 H I E V E D *************')
    1 FORMAT(//,2X,'NUMBER OF DEGREE OF FREEDOM = ',I2,/,2X,'NUMBER OF N
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     1ODE PER ELEMENT   ',I2,/,2X,'NUMBER OF ELEMENT IN THE MESH = ',I2,DGN04410
     2/,2X,'NUMBER OF NODE IN THIS MESH = ',I2,//,2X,'STRUCTURE DIMENSIODGN04420
     1N : XL   ',E13.5,'    YL = ',E13.5,//,3X,' R A D I U S   X - D I R DGN04430
     2E C T I O N = ',E13.5,/,3X,' R A D I U S  Y - D I R E C T I O N = DGN04440
     2',E13.5,/)                                                        DGN04450
    5 FORMAT(/,' ..................M A T E R I A L   P R O P E R I T Y .DGN04460
     1.....................',/,2X,'E1 = ',E13.5,/,2X,'E2 = ',E13.5,/,2X, DGN04470
     1'G12  = ',E13.5,/,2X,'G13 = ',E13.5,/,2X,'G23 = ',E13.5,/,2X,     DGN04480
     1'ANU12 = ',E13.5,//)                                              DGN04490
   11 FORMAT(//,2X,'NUMBER OF LAYER = ',I2,/,2X,'ANGLE OF EACH LAYER = 'DGN04500
     1,/,2X,7(E13.5,2X),//)                                             DGN04510
   12 FORMAT(2X,'************ T H I C K N E S S = ',E14.5,              DGN04520
     2' ************'//,2X,'ERROR DETECTION ',E13.5,//)                 DGN04530
   16 FORMAT(//,2X,'..................C O O R D I N A T E   O F   E A C HDGN04540
     1  N O D E.....................',/,8X,'X',17X,'Y',17X,'Z',/,30(/,2X,DGN04550
     13(E13.5,2X)),/)                                                   DGN04560
   21 FORMAT(//2X,'BOOLEAN MATRIX'//10(9(2X,I2)/))                      DGN04570
   10 FORMAT(8F10.5)                                                    DGN04580
   20 FORMAT(9I5)                                                       DGN04590
   22 FORMAT(16I5)                                                      DGN04600
  320 FORMAT(///,5X,'***********************************************',/,5X,DGN04610
     1'*',42X,'*',/,5X,'*      L I N E A R  S O L U T I O N       *',  DGN04620
     2/,5X,'*',42X,'*',/,5X,'*****************************************DGN04630
     3**',//)                                                           DGN04640
  330 FORMAT(5X,'********* U - D I S P L A C E M E N T ***********',/)  DGN04650
  350 FORMAT(5X,'********* V - D I S P L A C E M E N T ***********',/)  DGN04660
  360 FORMAT(5X,'********* W - D I S P L A C E M E N T ***********',/)  DGN04670
  370 FORMAT(5X,'********* S L O P E - X ***********',/)                DGN04680
  380 FORMAT(5X,'********* S L O P E - Y ***********',/)                DGN04690
  340 FORMAT(6(3X,E13.5))                                               DGN04700
  530 FORMAT(//,2X,'N O.  O F   I T E R A T I O N = ',I2,/,2X,'E R R O R DGN04710
     1 = ',E13.5)                                                       DGN04720
  550 FORMAT(///,'  ************ L O A D  ',E13.5,' ************',///)  DGN04730
  560 FORMAT(//,2X,'*** N O.  O F  B O U N D A R Y  C O N D I T I O N = DGN04740
     1',I2,/)                                                           DGN04750
  570 FORMAT(//,2X,'**** NUMBER OF INTEGRATING GAUSSIAN POINT (NGP : BENDGN04760
     1DING TERMS.   LGP : SHEAR TERMS) **** ',/,2X,' N G P = ',I2,'   L DGN04770
     2G P = ',I2,/)                                                     DGN04780
  590 FORMAT(////,6X,'**SIGMX**',6X,'**SIGMY**',6X,'**SIGMZ**',6X,'**SIGDGN04790
     1MXY*',6X,'**SIGMXZ*',6X,'**SIGMYZ*'//)                            DGN04800
  600 FORMAT(2X,'@@@@@@@@@@@@@@@@@@@@@@@@@@@@',/,2X,'@ ELEMENT NO. = ',I2,5XDGN04810
     1,'@',/,2X,'@@@@@@@@@@@@@@@@@@@@@@@@@@@@',/)                       DGN04820
  610 FORMAT(7E13.5)                                                    DGN04830
  620 FORMAT(10I5)                                                      DGN04840
      STOP                                                              DGN04850
      END                                                               DGN04860
      SUBROUTINE DEFE(C,B,A,D,NN)                                       DGN04870
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN04880
      DIMENSION AI(6),A(6,6),D(45),C(6,45),B(6)                         DGN04890
      DO 10 I=1,5                                                       DGN04900
      AI(I)=0.0                                                         DGN04910
      DO 10 J=1,5                                                       DGN04920
   10 AI(I)=AI(I)+A(I,J)*B(J)                                           DGN04930
      DO 20 I=1,NN                                                      DGN04940
      DO 20 J=1,5                                                       DGN04950
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   20 B(I)=B(I)-A1(J)*C(J,I)
      RETURN
      END
      SUBROUTINE DMASS(C1,A,C,NN)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(3,45),C(45,45)
      DO 1 I=1,NN
      DO 1 J=1,NN
      DO 1 K=1,3
      C(I,J)=C(I,J)+C1*A(K,I)*A(K,J)
    1 CONTINUE
      RETURN
      END
      SUBROUTINE DSTIFF(A,B,D,E,MB1,MB2,MD1,MD2,M,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION B(MB1,MB2),C(9,45),D(MD1,MD2),A(6,6),E(45,45)
      DO 10 I=1,5
      DO 10 J=1,N
      C(I,J)=0.0
      DO 10 K=1,5
   10 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      DO 20 I=1,N
      DO 20 J=1,N
      DO 20 K=1,5
   20 E(I,J)=E(I,J)+D(K,I)*C(K,J)
      RETURN
      END
      SUBROUTINE MXMULT(A,B,C,NN)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(9,45),B(9,45),C(45,45)
      DO 10 I=1,NN
      DO 10 J=1,NN
      DO 10 K=1,9
   10 C(I,J)=C(I,J)+A(K,I)*B(K,J)
      RETURN
      END
      SUBROUTINE STIFF(NN,NPE,NDF,NOE,E,PO,ITER,ILOAD,NP,IFLAG,NGP,IGP,
     1           CA1,CA3,A0,C1,C2,NIT,NIMAX,NA2,IPROB)
C.......................................................................
C
C     THIS SUBROUTINE GENERATES THE ELEMENTAL STIFFNESS MATRIX, MASS
C     MATRIX, FORCE VECTOR AND BALANCE FORCE VECTOR
C       WT.....WEIGHT FUNCTION MATRIX.
C       GAUSS..GAUSSIAN POINT MATRIX.
C       GJINV..INVERSE OF JACOBIAN MATRIX.
C       XV.....TRANSFORMATION MATRIX BETWEEN LOCAL AND GLOBAL COORDINATE
C       NOE....ELEMENT NUMBER.
C       H......MASS MATRIX.
C       SD.....RELATIONSHIP BETWEEN THE STRAIN AND DISPLACEMENT DERIVA-
C              TIVE.
C       DH1,DH2..RELATIONSHIP BETWEEN THE DISPLACEMENT DERIVATIVE AND
C              DISPLACEMENT.
C
C.......................................................................
      IMPLICIT REAL*8(A-H,O-Z)
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      COMMON/MATL/A(2,3,6,6),CM(9,6,6)                                  DGN05510
      COMMON XV(3,3)                                                    DGN05520
      COMMON/STIFF/DSTIF(45,45),ELP(45),V(45),ELQ(45),WO(45),W1(45)     DGN05530
     1             ,W2(45),VAC(50,3),VEL(50,3),VACO(50,3),VELO(50,3)    DGN05540
      COMMON/MESH/COOD(50,3),ELXYZ(9,3),NOD(9,9)                        DGN05550
      COMMON/SHP/SF(9),GJINV(3,3),DSF(2,9)                              DGN05560
      COMMON/ECV/V1(50,3),V2(50,3),V3(50,3),VA(50,3),VB(50,3),VC(50,3)  DGN05570
     1           ,V1A(50,3),V2B(50,3),V3C(50,3)                         DGN05580
      DIMENSION GAUSS(4,4),DJ1(9,45),DJ2(9,45),DH1(9,45),DH(3,3),DC1(3) DGN05590
     1        ,DH2(9,45),A1(6,6),A2(6,6),A3(6,6),SD(6,9),SD1(6,45),SD2  DGN05600
     2        (6,45),DC2(3),DB1(3),DB2(3),WT(4,4),ESTIF(9,45),SD3(6,45) DGN05610
     3        ,TD(6,9),H(45,45),DK1(3,45),DK2(3,45),CSTIF(45,45),HT(3), DGN05620
     4        HV(3)                                                     DGN05630
      COMMON/STR/GU(9),GV(9),S1(6),S2(6),S3(6),XU(3,3),XV(3,3),GT(9),   DGN05640
     1           GS(9),GY(3,3)                                          DGN05650
      DATA GAUSS/4*0.0D0,-.57735027D0,.57735027D0,2*0.0D0,-.77459667D0, DGN05660
     20.0D0,.77459667D0,0.0D0,-.86113631D0,                             DGN05670
     3-.33998104D0,.33998104D0,.86113631D0/                             DGN05680
      DATA WT/2.0D0,3*0.0D0,2*1.0D0,2*0.0D0,.55555555D0,.88888888D0,    DGN05690
     2.55555555D0,0.0D0,.34785485D0,2*.65214515D0,.34785485D0/          DGN05700
      DO 10 I=1,NN                                                      DGN05710
      ELP(I)=0.0                                                        DGN05720
      ELQ(I)=0.0                                                        DGN05730
      DO 10 J=1,NN                                                      DGN05740
      CSTIF(I,J)=0.0                                                    DGN05750
      DSTIF(I,J)=0.0                                                    DGN05760
      H(I,J)=0.0                                                        DGN05770
   10 CONTINUE                                                          DGN05780
      IF(IPROB.EQ.0) GO TO 530                                          DGN05790
      DO 24 NI=1,NGP                                                    DGN05800
      DO 24 NJ=1,NGP                                                    DGN05810
      XI=GAUSS(NI,NGP)                                                  DGN05820
      ETA=GAUSS(NJ,NGP)                                                 DGN05830
      CALL SHAPE(DET,ETA,XI,NPE,NOE,I)                                  DGN05840
      CNST=DET*WT(NI,NGP)*WT(NJ,NGP)                                    DGN05850
      CD1=CA1*CNST                                                      DGN05860
      CD2=CA3*CNST                                                      DGN05870
      DO 220 I=1,3                                                      DGN05880
      HT(I)=0.0                                                         DGN05890
      HV(I)=0.0                                                         DGN05900
      DB1(I)=0.0                                                        DGN05910
      DB2(I)=0.0                                                        DGN05920
      DC1(I)=0.0                                                        DGN05930
      DC2(I)=0.0                                                        DGN05940
      DO 220 J=1,NN                                                     DGN05950
      DK1(I,J)=0.0                                                      DGN05960
  220 DK2(I,J)=0.0                                                      DGN05970
C...............................................................DGN05980
C       VELOCITY = DB1+Z*DB2                                            DGN05990
C       ACCELERATION DC1+Z*DC2                                          DGN06000
C...............................................................DGN06010
      DO 57 I=1,NPE                                                     DGN06020
      ND=NOD(NOE,I)                                                     DGN06030
      II=NDF*(I-1)+1                                                    DGN06040
      DO 54 KI=1,3                                                      DGN06050
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      DB1(K1)=DB1(K1)+SF(I)*W1(II+K1-1)                                 DGN06060
      DC1(K1)=DC1(K1)+SF(I)*W2(II+K1-1)                                 DGN06070
      DB2(K1)=DB2(K1)+SF(I)*VEL(ND,K1)*0.5*T                            DGN06080
      DC2(K1)=DC2(K1)+SF(I)*VAC(ND,K1)*0.5*T                            DGN06090
   54 CONTINUE                                                          DGN06100
      DK1(1,II)=SF(I)                                                   DGN06110
      DK1(2,II+1)=SF(I)                                                 DGN06120
      DK1(3,II+2)=SF(I)                                                 DGN06130
      DK2(1,II+3)=SF(I)*V1(ND,1)*0.5*T                                  DGN06140
      DK2(1,II+4)=-SF(I)*V2(ND,1)*0.5*T                                 DGN06150
      DK2(2,II+3)=SF(I)*V1(ND,2)*0.5*T                                  DGN06160
      DK2(2,II+4)=-SF(I)*V2(ND,2)*0.5*T                                 DGN06170
      DK2(3,II+3)=SF(I)*V1(ND,3)*0.5*T                                  DGN06180
      DK2(3,II+4)=-SF(I)*V2(ND,3)*0.5*T                                 DGN06190
   57 CONTINUE                                                          DGN06200
      IF(IFLAG.NE.0) GO TO 62                                           DGN06210
C.....................................................................DGN06220
C     GENERATE THE MASS MATRIX.                                         DGN06230
C.....................................................................DGN06240
   51 CALL DMASS(CD1,DK1,H,NN)                                          DGN06250
      CALL DMASS(CD2,DK2,H,NN)                                          DGN06260
   62 DO 77 I=1,NPE                                                     DGN06270
      ND=NOD(NOE,I)                                                     DGN06280
      II=NDF*(I-1)+1                                                    DGN06290
      VA=V3(ND,1)-V3C(ND,1)                                             DGN06300
      VB=V3(ND,2)-V3C(ND,2)                                             DGN06310
      VC=V3(ND,3)-V3C(ND,3)                                             DGN06320
      HV(1)=HV(1)+SF(I)*(W0(II)-W(II))                                  DGN06330
      HV(2)=HV(2)+SF(I)*(W0(II+1)-W(II+1))                              DGN06340
      HV(3)=HV(3)+SF(I)*(W0(II+2)-W(II+2))                              DGN06350
      HI(1)=HI(1)+SF(I)*(VA)*0.5*T                                      DGN06360
      HI(2)=HI(2)+SF(I)*(VB)*0.5*T                                      DGN06370
   77 HI(3)=HI(3)+SF(I)*(VC)*0.5*T                                      DGN06380
C.....................................................................DGN06390
C     CALCULATE THE FORCE VECTOR BY THE INERTIA EFFECT.                 DGN06400
C.....................................................................DGN06410
      DO 97 I=1,NN                                                      DGN06420
      DO 97 J=1,3                                                       DGN06430
      ELQ(I)=ELQ(I)+CD1*(C1*DB1(J)*DK1(J,I)+C2*DC1(J)*DK1(J,I))         DGN06440
     1        +CD2*(C1*DB2(J)*DK2(J,I)+C2*DC2(J)*DK2(J,I))              DGN06450
     2        +A0*CD1*DK1(J,I)*HV(J)+A0*CD2*DK2(J,I)*HI(J)              DGN06460
   97 CONTINUE                                                          DGN06470
   24 CONTINUE                                                          DGN06480
C.....................................................................DGN06490
C     CACULATE THE STIFFNESS MATRIX AND FORCE VECTOR                    DGN06500
C.....................................................................DGN06510
  530 DO 820 NEP=LGP,NGP                                                DGN06520
      DO 20 NI=1,NEP                                                    DGN06530
      DO 20 NJ=1,NEP                                                    DGN06540
      XI=GAUSS(NI,NEP)                                                  DGN06550
      ETA=GAUSS(NJ,NEP)                                                 DGN06560
      CALL SHAPE(DET,ETA,XI,NPE,NOE,I)                                  DGN06570
      CNST=DET*WT(NI,NEP)*WT(NJ,NEP)                                    DGN06580
      NW=1                                                              DGN06590
      IF(NEP.EQ.LGP.AND.LGP.NE.NGP) NW=2                                DGN06600
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      DO 460 K=1,5                                                      DGN06610
      DO 460 I=1,5                                                      DGN06620
      A1(K,I)=A(NW,1,K,I)*CNST                                          DGN06630
      A2(K,I)=A(NW,2,K,I)*CNST                                          DGN06640
      A3(K,I)=A(NW,3,K,I)*CNST                                          DGN06650
  460 CONTINUE                                                          DGN06660
      CALL STRAIN(NOE,NDF,1,NPE)                                        DGN06670
      DO 210 I=1,5                                                      DGN06680
      DO 210 J=1,NN                                                     DGN06690
      DJ1(I,J)=0.0                                                      DGN06700
      DJ2(I,J)=0.0                                                      DGN06710
      DH1(I,J)=0.0                                                      DGN06720
      DH2(I,J)=0.0                                                      DGN06730
  210 CONTINUE                                                          DGN06740
      DO 22 I=1,NPE                                                     DGN06750
      ND=NOD(NOE,I)                                                     DGN06760
      MA=NDF*(I-1)+1                                                    DGN06770
      CALL ZERO(DH)                                                     DGN06780
      DH(1,1)=DSF(1,I)                                                  DGN06790
      DH(2,1)=DSF(2,I)                                                  DGN06800
      DH(3,1)=0.0                                                       DGN06810
      CALL MXMUT2(GJINV,XV,XU,DH)                                       DGN06820
      CALL DISTR(DH,DH1,MA,0)                                           DGN06830
      CALL ZERO(DH)                                                     DGN06840
      DH(1,2)=DSF(1,I)                                                  DGN06850
      DH(2,2)=DSF(2,I)                                                  DGN06860
      DH(3,2)=0.0                                                       DGN06870
      CALL MXMUT2(GJINV,XV,XU,DH)                                       DGN06880
      CALL DISTR(DH,DH1,MA,1)                                           DGN06890
      CALL ZERO(DH)                                                     DGN06900
      DH(1,3)=DSF(1,I)                                                  DGN06910
      DH(2,3)=DSF(2,I)                                                  DGN06920
      DH(3,3)=0.0                                                       DGN06930
      CALL MXMUT2(GJINV,XV,XU,DH)                                       DGN06940
      CALL DISTR(DH,DH1,MA,2)                                           DGN06950
      CALL ZERO(DH)                                                     DGN06960
      DH(1,1)=DSF(1,I)*V1(ND,1)*0.5*T                                   DGN06970
      DH(1,2)=DSF(1,I)*V1(ND,2)*0.5*T                                   DGN06980
      DH(2,1)=DSF(2,I)*V1(ND,1)*0.5*T                                   DGN06990
      DH(2,2)=DSF(2,I)*V1(ND,2)*0.5*T                                   DGN07000
      DH(1,3)=DSF(1,I)*V1(ND,3)*0.5*T                                   DGN07010
      DH(2,3)=DSF(2,I)*V1(ND,3)*0.5*T                                   DGN07020
      CALL MXMUT2(GJINV,XV,XU,DH)                                       DGN07030
      CALL DISTR(DH,DH2,MA,3)                                           DGN07040
      CALL ZERO(DH)                                                     DGN07050
      DH(1,1)=-DSF(1,I)*V2(ND,1)*0.5*T                                  DGN07060
      DH(1,2)=-DSF(1,I)*V2(ND,2)*0.5*T                                  DGN07070
      DH(2,1)=-DSF(2,I)*V2(ND,1)*0.5*T                                  DGN07080
      DH(2,2)=-DSF(2,I)*V2(ND,2)*0.5*T                                  DGN07090
      DH(1,3)=-DSF(1,I)*V2(ND,3)*0.5*T                                  DGN07100
      DH(2,3)=-DSF(2,I)*V2(ND,3)*0.5*T                                  DGN07110
      CALL MXMUT2(GJINV,XV,XU,DH)                                       DGN07120
      CALL DISTR(DH,DH2,MA,4)                                           DGN07130
      CALL ZERO(DH)                                                     DGN07140
      DH(3,1)=SF(I)*V1(ND,1)*0.5*T                                      DGN07150
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      DH(3,2)=SI(I)*V1(ND,2)*0.5*T                                   DGN07160
      DH(3,3)=SI(I)*V1(ND,3)*0.5*T                                   DGN07170
      CALL MXMUL2(GJINV,XV,XU,DH)                                    DGN07180
      CALL DISTR(DH,DH1,HA,3)                                        DGN07190
      CALL ZERO(DH)                                                  DGN07200
      DH(3,1)=-SI(I)*V2(ND,1)*0.5*T                                  DGN07210
      DH(3,2)=-SI(I)*V2(ND,2)*0.5*T                                  DGN07220
      DH(3,3)=-SI(I)*V2(ND,3)*0.5*T                                  DGN07230
      CALL MXMUL2(GJINV,XV,XU,DH)                                    DGN07240
      CALL DISTR(DH,DH1,HA,4)                                        DGN07250
      CALL ZERO(DH)                                                  DGN07260
   22 CONTINUE                                                       DGN07270
      DO 63 I=1,6                                                    DGN07280
      DO 63 J=1,9                                                    DGN07290
      SD(I,J)=0.0                                                    DGN07300
      TD(I,J)=0.0                                                    DGN07310
   63 CONTINUE                                                       DGN07320
      SD(1,1)=1.+GU(1)                                               DGN07330
      SD(1,4)=GU(4)                                                  DGN07340
      SD(1,7)=GU(7)                                                  DGN07350
      SD(2,2)=GU(2)                                                  DGN07360
      SD(2,5)=GU(5)+1.                                               DGN07370
      SD(2,8)=GU(8)                                                  DGN07380
      SD(3,1)=GU(2)                                                  DGN07390
      SD(3,2)=GU(1)+1.                                               DGN07400
      SD(3,4)=GU(5)+1.                                               DGN07410
      SD(3,5)=GU(4)                                                  DGN07420
      SD(3,7)=GU(8)                                                  DGN07430
      SD(3,8)=GU(7)                                                  DGN07440
      SD(4,1)=GU(3)                                                  DGN07450
      SD(4,3)=GU(1)+1.                                               DGN07460
      SD(4,4)=GU(6)                                                  DGN07470
      SD(4,6)=GU(4)                                                  DGN07480
      SD(4,7)=GU(9)+1.                                               DGN07490
      SD(4,9)=GU(7)                                                  DGN07500
      SD(5,2)=GU(3)                                                  DGN07510
      SD(5,3)=GU(2)                                                  DGN07520
      SD(5,5)=GU(6)                                                  DGN07530
      SD(5,6)=GU(5)+1.                                               DGN07540
      SD(5,8)=GU(9)+1.                                               DGN07550
      SD(5,9)=GU(8)                                                  DGN07560
      TD(1,1)=GS(1)                                                  DGN07570
      TD(1,4)=GS(4)                                                  DGN07580
      TD(1,7)=GS(7)                                                  DGN07590
      TD(2,2)=GS(2)                                                  DGN07600
      TD(2,5)=GS(5)                                                  DGN07610
      TD(2,8)=GS(8)                                                  DGN07620
      TD(3,1)=GS(2)                                                  DGN07630
      TD(3,2)=GS(1)                                                  DGN07640
      TD(3,4)=GS(5)                                                  DGN07650
      TD(3,5)=GS(4)                                                  DGN07660
      TD(3,7)=GS(8)                                                  DGN07670
      TD(3,8)=GS(7)                                                  DGN07680
      TD(4,1)=GS(3)                                                  DGN07690
      TD(4,3)=GS(1)                                                  DGN07700
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      ID(4,4)=GS(6)                                                     DGN07710
      ID(4,6)=SS(4)                                                     DGN07720
      ID(4,7)=GS(9)                                                     DGN07730
      ID(4,9)=GS(7)                                                     DGN07740
      ID(5,2)=GS(3)                                                     DGN07750
      ID(5,3)=GS(2)                                                     DGN07760
      ID(5,5)=GS(6)                                                     DGN07770
      ID(5,6)=GS(5)                                                     DGN07780
      ID(5,8)=GS(9)                                                     DGN07790
      ID(5,9)=GS(8)                                                     DGN07800
      DO 60 I=1,3                                                       DGN07810
      II=3*(I-1)                                                        DGN07820
      DO 60 J=1,3                                                       DGN07830
      DO 60 L=1,NN                                                      DGN07840
      DO 60 K=1,3                                                       DGN07850
      LL=II+K                                                           DGN07860
      DJ1(II+J,L)=DJ1(II+J,L)+GJINV(J,K)*DH1(LL,L)                      DGN07870
   60 DJ2(II+J,L)=DJ2(II+J,L)+GJINV(J,K)*DH2(LL,L)                      DGN07880
C.......................................................................DGN07890
C     SD1,SD2,SD3...RELATION BETWEEN STRAIN AND INCREMENTAL DISPLACEMENTDGN07900
C.......................................................................DGN07910
      DO 70 I=1,5                                                       DGN07920
      DO 70 J=1,NN                                                      DGN07930
      SD1(I,J)=0.0                                                      DGN07940
      SD2(I,J)=0.0                                                      DGN07950
      SD3(I,J)=0.0                                                      DGN07960
      DO 70 K=1,9                                                       DGN07970
      SD1(I,J)=SD1(I,J)+SD(I,K)*DH1(K,J)                                DGN07980
      SD2(I,J)=SD2(I,J)+SD(I,K)*DH2(K,J)                                DGN07990
      SD2(I,J)=SD2(I,J)+ID(I,K)*DH1(K,J)                                DGN08000
      SD3(I,J)=SD3(I,J)+ID(I,K)*DH2(K,J)                                DGN08010
   70 CONTINUE                                                          DGN08020
      IF(IFLAG.NE.0)GO TO 96                                            DGN08030
      CALL DSTIFF(A1,SD1,SD1,DSTIF,6,NN,6,NN,6,NN)                      DGN08040
      IF(NA2.EQ.1)CALL DSTIFF(A2,SD1,SD2,DSTIF,6,NN,6,NN,6,NN)          DGN08050
      IF(NA2.EQ.1)CALL DSTIFF(A2,SD2,SD1,DSTIF,6,NN,6,NN,6,NN)          DGN08060
      CALL DSTIFF(A3,SD2,SD2,DSTIF,6,NN,6,NN,6,NN)                      DGN08070
   96 CNTINUE                                                           DGN08080
  721 IF(IFLAG.NE.0) GO TO 85                                           DGN08090
      DO 495 I=1,9                                                      DGN08100
      DO 495 J=1,NN                                                     DGN08110
  495 ESTIF(I,J)=0.0                                                    DGN08120
      CALL DSTF3(DH1,S1,A1,ESTIF,NN)                                    DGN08130
      IF(NA2.EQ.1) CALL DSTF3(DH1,S2,A2,ESTIF,NN)                       DGN08140
      IF(NA2.EQ.1)CALL DSTF3(DH2,S1,A2,ESTIF,NN)                        DGN08150
      CALL DSTF3(DH2,S2,A3,ESTIF,NN)                                    DGN08160
      CALL MXMULT(DH1,ESTIF,CSTIF,NN)                                   DGN08170
      DO 505 I=1,9                                                      DGN08180
      DO 505 J=1,NN                                                     DGN08190
  505 ESTIF(I,J)=0.0                                                    DGN08200
      IF(NA2.EQ.1)CALL DSTF3(DH1,S1,A2,ESTIF,NN)                        DGN08210
      CALL DSTF3(DH1,S2,A3,ESTIF,NN)                                    DGN08220
      CALL DSTF3(DH2,S1,A3,ESTIF,NN)                                    DGN08230
      CALL MXMULT(DH2,ESTIF,CSTIF,NN)                                   DGN08240
   85 CALL DEFF(SD1,S1,A1,EEP,NN)                                       DGN08250
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      IF(NA2.EQ.1)CALL DEFF(SD1,S2,A2,ELP,NN)                         DGN08260
      IF(NA2.EQ.1)CALL DEFF(SD2,S1,A2,ELP,NN)                         DGN08270
      CALL DEFF(SD2,S2,A3,ELP,NN)                                      DGN08280
   20 CONTINUE                                                         DGN08290
  420 CONTINUE                                                         DGN08300
      IF(LFLAG.NE.0)GO TO 521                                          DGN08310
      IF(NOL.NE.1)GO TO 313                                            DGN08320
  313 DO 310 I=1,NN                                                    DGN08330
      DO 310 J=1,NN                                                    DGN08340
  310 DSTIF(I,J)=DSTIF(I,J)+A0*H(I,J)+CSTIF(I,J)                       DGN08350
  521 IF(ILOAD.EQ.0) GO TO 610                                         DGN08360
      DO 213 NI=1,NGP                                                  DGN08370
      DO 213 NJ=1,NGP                                                  DGN08380
      XI=GAUSS(NI,NGP)                                                 DGN08390
      ETA=GAUSS(NJ,NGP)                                                DGN08400
      CALL SHAPE(DET,ETA,XI,NPE,NOL,1.0D0)                             DGN08410
      CNST=DET*WT(NI,NGP)*WT(NJ,NGP)                                   DGN08420
   86 DO 305 I=1,NPE                                                   DGN08430
      NE=NOD(NOL,I)                                                    DGN08440
      P1=P0*V3(NE,1)                                                   DGN08450
      P2=P0*V3(NE,2)                                                   DGN08460
      P3=P0*V3(NE,3)                                                   DGN08470
      L=(I-1)*NDF+1                                                    DGN08480
      ELQ(L)=ELQ(L)+CNST*SF(I)*P1*2.0                                  DGN08490
      ELQ(L+1)=ELQ(L+1)+CNST*SF(I)*P2*2.0                              DGN08500
  305 ELQ(L+2)=ELQ(L+2)+CNST*SF(I)*P3*2.0                              DGN08510
  213 CONTINUE                                                         DGN08520
  610 CONTINUE                                                         DGN08530
      RETURN                                                           DGN08540
      END                                                              DGN08550
      SUBROUTINE STRESS(NGP,NOL,L,NN,Z,NPE,LAYER)                      DGN08560
      IMPLICIT REAL*8(A-H,O-Z)                                         DGN08570
      COMMON/MATL/A(2,3,6,6),CH(9,6,6)                                 DGN08580
      COMMON XV(3,3)                                                   DGN08590
      COMMON/STIF/DSTIF(45,45),ELP(45),W(45),ELQ(45),W0(45),V1(45)     DGN08600
     1           ,W2(45),VAC(50,3),VEL(50,3),VAC0(50,3),VEL0(50,3)     DGN08610
      COMMON/MESH/COOD(50,3),ELXYZ(9,3),NOD(9,9)                       DGN08620
      COMMON/SHP/SF(9),GJINV(3,3),DSF(2,9)                             DGN08630
      COMMON/ECV/V1(50,3),V2(50,3),V3(50,3),VA(50,3),VB(50,3),VC(50,3) DGN08640
     1          ,V1A(50,3),V2B(50,3),V3C(50,3)                         DGN08650
      DIMENSION GAUSS(4,4),Z(10),S1(5),SB(5),IT(5),IB(5)               DGN08660
      COMMON/STR/GU(9),GV(9),S1(6),S2(6),S3(6),XU(3,3),XV(3,3),GT(9),  DGN08670
                 GS(9),GY(3,3)                                         DGN08680
      DATA GAUSS/4*0.0D0,-.5773502700,.5773502700,2*0.0D0,-.7745966700,DGN08690
     20.0D0,.7745966700,0.0D0,-.8611363100,                            DGN08700
     3-.3399810400,.3399810400,.8611363100/                            DGN08710
      WRITE(6,10)                                                      DGN08720
      DO 20 NI=1,1                                                     DGN08730
      DO 20 NJ=1,1                                                     DGN08740
      XI=GAUSS(NI,NGP)                                                 DGN08750
      ETA=GAUSS(NJ,NGP)                                                DGN08760
      CALL SHAPE(DET,ETA,XI,NPE,NOL,1)                                 DGN08770
      X=0.0                                                            DGN08780
      Y=0.0                                                            DGN08790
      ZZ=0.0                                                           DGN08800
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      DO 220 I=1,NPE                                                    DGN08810
      X=X+ELXYZ(I,1)*SF(I)                                              DGN08820
      Y=Y+ELXYZ(I,2)*SF(I)                                              DGN08830
      ZZ=ZZ+ELXYZ(I,3)*SF(I)                                            DGN08840
  220 CONTINUE                                                          DGN08850
      WRITE(6,230)X,Y,ZZ                                                DGN08860
  230 FORMAT(2X,'*** X- COOD ***',3X,'*** Y- COOD ***',3X,'*** Z- COOD *DGN08870
     1**',/,2X,3(E13.5,2X))                                             DGN08880
      CALL STRAIN(NOE,NDF,I,NPE)                                        DGN08890
      DO 20 I=1,LAYER                                                   DGN08900
      DO 30 II=1,5                                                      DGN08910
      ST(II)=0.0                                                        DGN08920
   30 SB(II)=0.0                                                        DGN08930
      DO 35 II=1,5                                                      DGN08940
      TT(II)=S1(II)+Z(I)*S2(II)+Z(I)**2*S3(II)                          DGN08950
   35 TB(II)=S1(II)+Z(I+1)*S2(II)+Z(I+1)**2*S3(II)                      DGN08960
      DO 37 M1=1,5                                                      DGN08970
      DO 37 M2=1,5                                                      DGN08980
      ST(M1)=ST(M1)+CM(I,M1,M2)*TT(M2)                                  DGN08990
   37 SB(M1)=SB(M1)+CM(I,M1,M2)*TB(M2)                                  DGN09000
      WRITE(6,15)(ST(II),II=1,5)                                        DGN09010
      WRITE(6,15)(SB(II),II=1,5)                                        DGN09020
   20 CONTINUE                                                          DGN09030
   15 FORMAT(2X,6(E13.5,1X))                                            DGN09040
   10 FORMAT(/,' ********************************************************DGN09050
     1*********************************************',/)                 DGN09060
      RETURN                                                            DGN09070
      END                                                               DGN09080
      SUBROUTINE SHAPE(DET,ETA,XI,NPE,NOE,II)                           DGN09090
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN09100
C     THIS SUBROUTINE EVALUATES THE SHAPE FUNCTIONS AND THEIR DERIVATIVEDGN09110
C     THE GUASSIAN POINTS (ISOPARAMETRIC QUADRILATERAL ELEMENT WITH NINEDGN09120
C     NODES).                                                           DGN09130
      COMMON/MESH/COOD(50,3),ELXYZ(9,3),NOD(9,9)                        DGN09140
      COMMON/ECV/V1(50,3),V2(50,3),V3(50,3),VA(50,3),VB(50,3), ?3(50,3) DGN09150
     1          ,V1A(50,3),V2B(50,3),V3C(50,                            DGN09160
      COMMON XV(3,3)                                                    DGN09170
      COMMON/SHP/SF(9),GJINV(3,3),DSF(2,9)                              DGN09180
      DIMENSION XNODE(9,2),NP(9),GJ(3,3),WKA  A(3)                      DGN09190
      DATA XNODE/-1.0D0,2*1.0D0,-1.0D0,0.0D0,1.0D0,0.0D0,-1.0D0,0.0D0,  DGN09200
     C           2*-1.0D0,2*1.0D0,-1.0D0,0.0D0,1.0D0,2*0.0D0/           DGN09210
      DATA NP/1,2,3,4,5,7,6,8,9/                                        DGN09220
      FNC(A,B)=A*B                                                      DGN09230
      IF (NPE-8) 50,5,65                                                DGN09240
    5 DO 40 I=1, NPE                                                    DGN09250
      NI=NP(I)                                                          DGN09260
      XP=XNODE(NI,1)                                                    DGN09270
      YP=XNODE(NI,2)                                                    DGN09280
      XI0=1.0+XI*XP                                                     DGN09290
      ETA0=1.0+ETA*YP                                                   DGN09300
      XI1=1.0-XI*XI                                                     DGN09310
      ETA1=1.0-ETA*ETA                                                  DGN09320
      IF(I.GT.4) GO TO 10                                               DGN09330
      SF(NI)=0.25*FNC(XI0,ETA0)*(XI*XP+ETA*YP-1.0)                      DGN09340
      DSF(1,NI)=0.25*FNC(ETA0,XP)*(2.0*XI*XP+ETA*YP)                    DGN09350
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      DSI(2,NI) = 0.25*FNC(X10,YP)*(2.0*ETA*YP+XI*XP)                DGN09360
      GO TO 40                                                       DGN09370
   10 IF(I.GT.6) GO TO 20                                            DGN09380
      SI(NI) = 0.5*FNC(X11,ETA0)                                     DGN09390
      DSI(1,NI) = -FNC(XI,ETA0)                                      DGN09400
      DSI(2,NI) = 0.5*FNC(YP,X11)                                    DGN09410
      GO TO 40                                                       DGN09420
   20 SI(NI) = 0.5*FNC(ETA1,X10)                                     DGN09430
      DSI(1,NI) = 0.5*FNC(XP,ETA1)                                   DGN09440
      DSI(2,NI) = -FNC(ETA,X10)                                      DGN09450
   40 CONTINUE                                                       DGN09460
      GO TO 70                                                       DGN09470
   50 CONTINUE                                                       DGN09480
      DO 60 I = 1, NPI                                               DGN09490
      XP=XNODE(I,1)                                                  DGN09500
      YP=XNODE(I,2)                                                  DGN09510
      X10=1.0+XI*XP                                                  DGN09520
      ETA0=1.0+ETA*YP                                                DGN09530
      SI(I)=0.25*FNC(X10,ETA0)                                       DGN09540
      DSI(1,I)=0.25*FNC(XP,ETA0)                                     DGN09550
      DSI(2,I)=0.25*FNC(YP,X10)                                      DGN09560
   60 CONTINUE                                                       DGN09570
      GO TO 70                                                       DGN09580
   65 DO 69 I=1,NPI                                                  DGN09590
      NI = NP(I)                                                     DGN09600
      XP = XNODE(NI,1)                                               DGN09610
      YP = XNODE(NI,2)                                               DGN09620
      X10 = 1.0+XI*XP                                                DGN09630
      ETA0 = 1.0+ETA*YP                                              DGN09640
      X11 = 1.0-XI*XI                                                DGN09650
      ETA1 = 1.0-ETA*ETA                                             DGN09660
      X12 = XP*XI                                                    DGN09670
      ETA2 = YP*ETA                                                  DGN09680
      IF(I .GT. 4)GOTO 66                                            DGN09690
      SI(NI) = 0.25*FNC(X10,ETA0)*X12*ETA2                           DGN09700
      DSI(1,NI)=0.25*XP*FNC(ETA2,ETA0)*(1.0+2.0*X12)                 DGN09710
      DSI(2,NI)=0.25*YP*FNC(X12,X10)*(1.0+2.0*ETA2)                  DGN09720
      GO TO 69                                                       DGN09730
   66 IF(I .GT. 6)GO TO 67                                           DGN09740
      SI(NI) = 0.5*FNC(X11,ETA0)*ETA2                                DGN09750
      DSI(1,NI) = -XI*FNC(ETA2,ETA0)                                 DGN09760
      DSI(2,NI) = 0.5*FNC(X11,YP)*(1.0+2.0*ETA2)                     DGN09770
      GO TO 69                                                       DGN09780
   67 IF(I .GT. 8)GO TO 68                                           DGN09790
      SI(NI) = 0.5*FNC(ETA1,X10)*X12                                 DGN09800
      DSI(2,NI) = -ETA*FNC(X12,X10)                                  DGN09810
      DSI(1,NI) = 0.5*FNC(ETA1,XP)*(1.0+2.0*X12)                     DGN09820
      GO TO 69                                                       DGN09830
   68 SI(NI) = FNC(X11,ETA1)                                         DGN09840
      DSI(1,NI) = -2.0*XI*ETA1                                       DGN09850
      DSI(2,NI) = -2.0*ETA*X11                                       DGN09860
   69 CONTINUE                                                       DGN09870
   70 DO 80 I = 1,3                                                  DGN09880
      DO 80 J = 1,3                                                  DGN09890
      GJ(I,J) = 0.0                                                  DGN09900
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      DO 80 K = 1,NPE                                                   DGN09910
      IF (I.EQ.3) GO TO 90                                              DGN09920
      GJ(I,J) = GJ(I,J) + DSF(I,K)*ELXYZ(K,J)                           DGN09930
      GO TO 80                                                          DGN09940
   90 NI=NOD(NOE,K)                                                     DGN09950
      GJ(I,J)=GJ(I,J)+SF(K)*VC(NI,J)*0.5*H                              DGN09960
   80 CONTINUE                                                          DGN09970
      XV(1,3)=GJ(1,2)*GJ(2,3)-GJ(2,2)*GJ(1,3)                           DGN09980
      XV(2,3)=GJ(2,1)*GJ(1,3)-GJ(1,1)*GJ(2,3)                           DGN09990
      XV(3,3)=GJ(1,1)*GJ(2,2)-GJ(2,1)*GJ(1,2)                           DGN10000
      XV(1,1)=XV(3,3)                                                   DGN10010
      XV(2,1)=0.0                                                       DGN10020
      XV(3,1)=-XV(1,3)                                                  DGN10030
      XV(1,2)=-XV(2,3)*XV(1,3)                                          DGN10040
      XV(2,2)=XV(1,3)**2+XV(3,3)**2                                     DGN10050
      XV(3,2)=-XV(2,3)*XV(3,3)                                          DGN10060
      SUM1=DSQRT(XV(1,1)**2+XV(2,1)**2+XV(3,1)**2)                      DGN10070
      SUM2=DSQRT(XV(1,2)**2+XV(2,2)**2+XV(3,2)**2)                      DGN10080
      SUM3=DSQRT(XV(1,3)**2+XV(2,3)**2+XV(3,3)**2)                      DGN10090
      DO 30 J=1,3                                                       DGN10100
      XV(J,1)=XV(J,1)/SUM1                                              DGN10110
      XV(J,2)=XV(J,2)/SUM2                                              DGN10120
      XV(J,3)=XV(J,3)/SUM3                                              DGN10130
   30 CONTINUE                                                          DGN10140
      DET=   GJ(1,1)*GJ(2,2)*GJ(3,3)+GJ(1,2)*GJ(2,3)*GJ(3,1)+GJ(2,1)*   DGN10150
     1     GJ(3,2)*GJ(1,3)-GJ(1,3)*GJ(2,2)*GJ(3,1)-GJ(2,3)*GJ(3,2)*GJ   DGN10160
     2     (1,1)-GJ(1,2)*GJ(2,1)*GJ(3,3)                                DGN10170
      CALL LINV1F(GJ,3,3,GJINV,0,WKAREA,IER)                            DGN10180
      RETURN                                                            DGN10190
      END                                                               DGN10200
      SUBROUTINE DSTF3(D2,B,A,A2,NN)                                    DGN10210
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN10220
      DIMENSION D2(9,45),A(6,6),A1(6),B1(9,9),                          DGN10230
     1          A2(9,45),B(6)                                           DGN10240
      DO 10 I=1,5                                                       DGN10250
      A1(I)=0.0                                                         DGN10260
      DO 10 J=1,5                                                       DGN10270
   10 A1(I)=A1(I)+A(I,J)*B(J)                                           DGN10280
      DO 5 I=1,9                                                        DGN10290
      DO 5 J=1,9                                                        DGN10300
    5 B1(I,J)=0.0                                                       DGN10310
      DO 20 I=1,3                                                       DGN10320
      II=3*(I-1)                                                        DGN10330
      JJ=3*(I-1)                                                        DGN10340
      B1(II+1,JJ+1)=A1(1)                                               DGN10350
      B1(II+1,JJ+2)=A1(3)                                               DGN10360
      B1(II+1,JJ+3)=A1(4)                                               DGN10370
      B1(II+2,JJ+1)=A1(3)                                               DGN10380
      B1(II+2,JJ+2)=A1(2)                                               DGN10390
      B1(II+2,JJ+3)=A1(5)                                               DGN10400
      B1(II+3,JJ+1)=A1(4)                                               DGN10410
      B1(II+3,JJ+2)=A1(5)                                               DGN10420
      B1(II+3,JJ+3)=0.0                                                 DGN10430
   20 CONTINUE                                                          DGN10440
      DO 30 II=1,3                                                      DGN10450
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      I2=3*(I1-1)                                                       DGN10460
      DO 30 I3=1,3                                                      DGN10470
      I2=I2+1                                                           DGN10480
      DO 30 J1=1,NN                                                     DGN10490
      K2=3*(I1-1)                                                       DGN10500
      DO 30 K1=1,3                                                      DGN10510
      K2=K2+1                                                           DGN10520
      A2(I2,J1)=A2(I2,J1)+B1(I2,K2)*D2(K2,J1)                           DGN10530
   30 CONTINUE                                                          DGN10540
      RETURN                                                            DGN10550
      END                                                               DGN10560
      SUBROUTINE BOU(NRMAX,A,D,DD,NEQ,NEQW,NBDY,IBDY,ITER,IFLAG)        DGN10570
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN10580
      DIMENSION A(NRMAX,NRMAX),D(NRMAX),IBDY(NBDY),DD(NRMAX)            DGN10590
  300 DO 2 I=1,NBDY                                                     DGN10600
      IMAX=IBDY(I)                                                      DGN10610
      DO 1 J=I,NBDY                                                     DGN10620
      IF (IBDY(J).LE.IMAX) GO TO 1                                      DGN10630
      IMAX=IBDY(J)                                                      DGN10640
      IKEPT=J                                                           DGN10650
    1 CONTINUE                                                          DGN10660
      IBDY(IKEPT)=IBDY(I)                                               DGN10670
      IBDY(I)=IMAX                                                      DGN10680
    2 CONTINUE                                                          DGN10690
      NEQW = NEQ                                                        DGN10700
      DO 14 I=1,NBDY                                                    DGN10710
    8 IB=IBDY(I)                                                        DGN10720
      IF (IB .GE. NEQW)GO TO 13                                         DGN10730
      NEQW1=NEQW-1                                                      DGN10740
      DO 12 II=IB,NEQW1                                                 DGN10750
      DO 9 JJ=1,NEQW                                                    DGN10760
      D(II)=D(II+1)                                                     DGN10770
      DD(II)=DD(II+1)                                                   DGN10780
      IF(IFLAG.NE.0) GO TO 9                                            DGN10790
      A(II,JJ)=A(II+1,JJ)                                               DGN10800
    9 CONTINUE                                                          DGN10810
      IF(IFLAG.NE.0)GO TO 12                                            DGN10820
      DO 10 JJ=1,NEQW                                                   DGN10830
   10 A(JJ,II)=A(JJ,II+1)                                               DGN10840
   12 CONTINUE                                                          DGN10850
   13 NEQW=NEQW-1                                                       DGN10860
   14 CONTINUE                                                          DGN10870
      RETURN                                                            DGN10880
      END                                                               DGN10890
      SUBROUTINE HESHEL(THA1,THA2,M1,M2,R,ITYPE,XL,YL)                  DGN10900
C                                                                       DGN10910
C........ THIS SUBROUTINE IS TO FIND THE GLOBAL COORDINATE OF A DOUBLY   DGN10920
C          CURVED SHELL WITH EQUAL RADIUS                               DGN10930
C                                                                       DGN10940
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN10950
      COMMON/HESHI/COOD(50,3),ELXYZ(9,3),NOD(9,9)                       DGN10960
      COMMON/LCV/V1(50,3),V2(50,3),V3(50,3),VA(50,3),VB(50,3),VC(50,3)  DGN10970
     1          ,V1A(50,3),V2B(50,3),V3C(50,3)                          DGN10980
      DIMENSION XTH(15),YTH(15)                                         DGN10990
C       ITYPE 1.... CYLINDRICAL SHELL                                   DGN11000
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C       ITYPE 2...  SPHERICAL SHELL
C
C ..................................................................

      N1=M1+1
      N2=M2+1
      IF(ITYPE.EQ.0)GO TO 100
      TH1=THA1/M1
      IF(ITYPE.NE.1) GO TO 10
      YL1=YL/M2
      GO TO 11
   10 TH2=THA2/M2
   11 THB=0.0
      DO 1 I=1,N1
      XTH(I)=R*DSIN(THB)
      THB=THB+TH1
    1 CONTINUE
      THB=0.0
      DO 2 I=1,N2
      IF(ITYPE.NE.1) GO TO 13
      YTH(I)=THB
      THB=THB+YL1
      GO TO 2
   13 YTH(I)=R*DSIN(THB)
      THB=THB+TH2
    2 CONTINUE
      II=1
      DO 3 I=1,N2
      DO 4 J=1,N1
      OCI=DSQRT(XTH(J)**2+YTH(I)**2)
      IF(ITYPE.EQ.1) OCI=XTH(J)
      PHI=DARSIN(OCI/R)
      IF(OCI.EQ.0) THE=0.0
      IF(OCI.EQ.0) GO TO 5
      IF(ITYPE.EQ.1) THE=3.14159625/2.0
      IF(ITYPE.EQ.1) GO TO 5
      THE=DARCOS(YTH(I)/OCI)
    5 COOD(II,1)=R*DSIN(PHI)*DSIN(THE)
      COOD(II,3)=R*DCOS(PHI)
      V3(II,1)=DSIN(PHI)*DSIN(THE)
      V3(II,3)=DCOS(PHI)
      IF(ITYPE.NE.1) GO TO 15
      V3(II,2)=0.0
      COOD(II,2)=YTH(I)
      GO TO 16
   15 V3(II,2)=DSIN(PHI)*DCOS(THE)
      COOD(II,2)=R*DSIN(PHI)*DCOS(THE)
   16 II=II+1
    4 CONTINUE
    3 CONTINUE
      RETURN
100   XL=XL/M1
      YL=YL/M2
      YL=YL/M2
      B=0.0
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      IC=1                                                               DGN11560
      DO 120 I=1,N2                                                      DGN11570
      A=0.0                                                              DGN11580
      DO 110 J=1,N1                                                      DGN11590
      COOD(IC,1)=A                                                       DGN11600
      COOD(IC,2)=B                                                       DGN11610
      COOD(IC,3)=0.0                                                     DGN11620
      V1(IC,1)=1.0                                                       DGN11630
      V1(IC,2)=0.0                                                       DGN11640
      V1(IC,3)=0.0                                                       DGN11650
      V2(IC,1)=0.0                                                       DGN11660
      V2(IC,2)=1.0                                                       DGN11670
      V2(IC,3)=0.0                                                       DGN11680
      V3(IC,1)=0.0                                                       DGN11690
      V3(IC,2)=0.0                                                       DGN11700
      V3(IC,3)=1.0                                                       DGN11710
      A=A+X1                                                             DGN11720
      IC=IC+1                                                            DGN11730
  110 CONTINUE                                                           DGN11740
      B=B+Y1                                                             DGN11750
  120 CONTINUE                                                           DGN11760
      RETURN                                                             DGN11770
      END                                                                DGN11780
      SUBROUTINE NXHOLE(A,C,B)                                           DGN11790
      IMPLICIT REAL*8(A-H,O-Z)                                           DGN11800
      DIMENSION Q(6,6),A(6,6),B(6,6),C(6,6),Q1(6,6),Q2(6,6),Q3(6)        DGN11810
     1         ,R(6,6),R1(6,6)                                           DGN11820
      DO 10 I=1,6                                                        DGN11830
      DO 10 J=1,6                                                        DGN11840
      Q1(I,J)=B(I,J)                                                     DGN11850
      R(I,J)=0.0                                                         DGN11860
      IF(I.GT.3)R(I,I)=2.0                                               DGN11870
      IF(I.LT.3)R(I,I)=1.0                                               DGN11880
   10 Q(I,J)=0.0                                                         DGN11890
      CALL LINV1F(Q1,6,6,Q2,0,Q3,IER)                                    DGN11900
      DO 100 I=1,6                                                       DGN11910
      DO 100 J=1,6                                                       DGN11920
      DO 100 K=1,6                                                       DGN11930
100   Q(I,J)=Q(I,J)+R(I,K)*Q2(K,J)                                       DGN11940
      CALL LINV1F(R,6,6,R1,0,Q3,IER)                                     DGN11950
      DO 200 I=1,6                                                       DGN11960
      DO 200 J=1,6                                                       DGN11970
      R(I,J)=0.0                                                         DGN11980
      DO 200 K=1,6                                                       DGN11990
200   R(I,J)=R(I,J)+Q(I,K)*R1(K,J)                                       DGN12000
      DO 20 I=1,6                                                        DGN12010
      DO 20 J=1,6                                                        DGN12020
      Q(I,J)=0.0                                                         DGN12030
      DO 20 K=1,6                                                        DGN12040
   20 Q(I,J)=Q(I,J)+A(I,K)*R(K,J)                                        DGN12050
      DO 25 I=1,6                                                        DGN12060
      DO 25 J=1,6                                                        DGN12070
   25 C(I,J)=0.0                                                         DGN12080
   30 DO 40 I=1,6                                                        DGN12090
      DO 40 J=1,6                                                        DGN12100
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      DO 40 K=1,6                                                       DGN12110
   40 C(I,J)=C(I,J)+B(I,K)*Q(K,J)                                       DGN12120
      RETURN                                                            DGN12130
      END                                                               DGN12140
      SUBROUTINE MATPRP(E1,E2,ANU12,G12,G13,G23,H,LAYER,NNM,THETA,Z,    DGN12150
     1                  NGP,IGP)                                        DGN12160
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN12170
      COMMON/MATL/A(2,3,6,6),CH(9,6,6)                                  DGN12180
      DIMENSION Z(10),T(10),QH(6,6),QS(6,6),THETA(10),A1(3),A2(3),      DGN12190
     1          A3(3),C(5,5),Q(5,5),Q1(5,5)                             DGN12200
      COMMON/ECV/V1(50,3),V2(50,3),V3(50,3),VA1(50,3),VB2(50,3),        DGN12210
     1VC3(50,3),VA(50,3),VB(50,3),VC(50,3)                              DGN12220
      AK=5./6.                                                          DGN12230
      PI=3.14159620D0                                                   DGN12240
      DO 10 I=1,LAYER                                                   DGN12250
   10 T(I)=2.0/LAYER                                                    DGN12260
      ZT=0.0                                                            DGN12270
      LH=(LAYER-1)/2                                                    DGN12280
      IF(LAYER/2*2.EQ.LAYER) LH=LAYER/2                                 DGN12290
      IF(LAYER/2*2.NE.LAYER)ZT=T(LH+1)*0.5                              DGN12300
      IF(LAYER.EQ.1)GO TO 30                                            DGN12310
      DO 20 I=1,LH                                                      DGN12320
   20 ZT=ZT+T(I)                                                        DGN12330
   30 Z(1)=-ZT                                                          DGN12340
      DO 40 I=1,LAYER                                                   DGN12350
   40 Z(I+1)=Z(I)+T(I)                                                  DGN12360
      DO 60 I=1,5                                                       DGN12370
      DO 60 J=1,5                                                       DGN12380
      C(I,J)=0.0                                                        DGN12390
      Q(I,J)=0.0                                                        DGN12400
      Q1(I,J)=0.0                                                       DGN12410
   60 CONTINUE                                                          DGN12420
      ANU21=E2*ANU12/E1                                                 DGN12430
      DENOM=1.-ANU12*ANU21                                              DGN12440
      C(1,1)=E1/DENOM                                                   DGN12450
      C(1,2)=ANU12*E2/DENOM                                             DGN12460
      C(2,1)=C(1,2)                                                     DGN12470
      C(2,2)=E2/DENOM                                                   DGN12480
      C(3,3)=G12                                                        DGN12490
      DO 50 II=1,2                                                      DGN12500
      DO 50 J=1,3                                                       DGN12510
      DO 50 K=1,6                                                       DGN12520
      DO 50 L=1,6                                                       DGN12530
   50 A(II,J,K,L)=0.0                                                   DGN12540
      DO 70 LA=1,LAYER                                                  DGN12550
      ANGLE=THETA(LA)/180.*PI                                           DGN12560
      CN=DCOS(ANGLE)                                                    DGN12570
      SN=DSIN(ANGLE)                                                    DGN12580
      A1(1)=CN                                                          DGN12590
      A1(2)=-SN                                                         DGN12600
      A1(3)=0.0                                                         DGN12610
      A2(1)=SN                                                          DGN12620
      A2(2)=CN                                                          DGN12630
      A2(3)=0.0                                                         DGN12640
      A3(1)=0.0                                                         DGN12650
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      A3(2)=0.0
      A3(3)=1.
      Q(1,1)=C(1,1)*(CN**4)+2.0*(C(1,2)+2.0*C(3,3))*SN*SN*CN*CN+C(2,2)
     1*(SN**4)
      Q(1,2)=(C(1,1)+C(2,2)-4.0*C(3,3))*SN*SN*CN*CN+C(1,2)*(SN**4+CN**
     14)
      Q(2,2)=C(1,1)*(SN**4)+2.0*(C(1,2)+2.0*C(3,3))*SN*SN*CN*CN+C(2,2)
     1*(CN**4)
      Q(1,3)=(C(1,1)-C(1,2)-2.0*C(3,3))*SN*(CN**3)+(C(1,2)-C(2,2)+2.0*
     1C(3,3))*(SN**3)*CN
      Q(2,3)=(C(1,1)-C(1,2)-2.0*C(3,3))*(SN**3)*CN+(C(1,2)-C(2,2)+2.0*
     1C(3,3))*SN*(CN**3)
      Q(3,3)=(C(1,1)+C(2,2)-2.0*C(1,2)-2.0*C(3,3))*SN*SN*CN*CN+C(3,3)*
     1(SN**4+CN**4)
      Q(2,1)=Q(1,2)
      Q(3,1)=Q(1,3)
      Q(3,2)=Q(2,3)
      CM(IA,4,4)=(G23*CN*CN+G13*SN*SN)*AK
      CM(IA,4,5)=((G23-G13)*CN*SN)*AK
      CM(IA,5,5)=(G13*CN*CN+G23*SN*SN)*AK
      IF(NGP.EQ.IGP)Q(4,4)=(G23*CN*CN+G13*SN*SN)*AK
      IF(NGP.EQ.IGP)Q(4,5)=((G23-G13)*CN*SN)*AK
      IF(NGP.EQ.IGP)Q(5,5)=(G13*CN*CN+G23*SN*SN)*AK
      IF(NGP.NE.IGP)Q1(4,4)=(G23*CN*CN+G13*SN*SN)*AK
      IF(NGP.NE.IGP)Q1(4,5)=((G23-G13)*CN*SN)*AK
      IF(NGP.NE.IGP)Q1(5,5)=(G13*CN*CN+G23*SN*SN)*AK
      DO 91 I=1,3
      DO 91 J=1,3
   91 CM(IA,I,J)=Q(I,J)
      DO 90 I=1,5
      DO 90 J=1,5
      A(1,1,I,J)=A(1,1,I,J)+Q(I,J)*(Z(IA+1)-Z(IA))
      A(1,2,I,J)=A(1,2,I,J)+Q(I,J)*(Z(IA+1)**2-Z(IA)**2)/2.0
      A(1,3,I,J)=A(1,3,I,J)+Q(I,J)*(Z(IA+1)**3-Z(IA)**3)/3.0
      A(2,1,I,J)=A(2,1,I,J)+Q1(I,J)*(Z(IA+1)-Z(IA))
      A(2,2,I,J)=A(2,2,I,J)+Q1(I,J)*(Z(IA+1)**2-Z(IA)**2)/2.0
      A(2,3,I,J)=A(2,3,I,J)+Q1(I,J)*(Z(IA+1)**3-Z(IA)**3)/3.0
   90 CONTINUE
   70 CONTINUE
      RETURN
      END
      SUBROUTINE MXMUL2 (GJINV,XV,XU,DH)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION GJINV(3,3),XV(3,3),XU(3,3),DH(3,3),DK(3,3)
      CALL MLPLY(DH,XV,DK)
      CALL MLPLY(GJINV,DK,DH)
      CALL MLPLY(XU,DH,DK)
      DO 1 I=1,3
      DO 1 J=1,3
    1 DH(I,J)=DK(I,J)
      RETURN
      END
      SUBROUTINE MLPLY(A,B,C)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(3,3),B(3,3),C(3,3)
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      DO 1 I=1,3                                                        DGN13210
      DO 1 J=1,3                                                        DGN13220
      C(I,J)=0.0                                                        DGN13230
      DO 1 K=1,3                                                        DGN13240
    1 C(I,J)=C(I,J)+A(I,K)*B(K,J)                                       DGN13250
      RETURN                                                            DGN13260
      END                                                               DGN13270
      SUBROUTINE ZERO(A)                                                DGN13280
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN13290
      DIMENSION A(3,3)                                                  DGN13300
      DO 1 I=1,3                                                        DGN13310
      DO 1 J=1,3                                                        DGN13320
    1 A(I,J)=0.0                                                        DGN13330
      RETURN                                                            DGN13340
      END                                                               DGN13350
      SUBROUTINE DISTR(A,B,I,J)                                         DGN13360
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN13370
      DIMENSION  A(3,3),B(9,45)                                         DGN13380
      B(1,I+J)=B(1,I+J)+A(1,1)                                          DGN13390
      B(2,I+J)=B(2,I+J)+A(2,1)                                          DGN13400
      B(3,I+J)=B(3,I+J)+A(3,1)                                          DGN13410
      B(4,I+J)=B(4,I+J)+A(1,2)                                          DGN13420
      B(5,I+J)=B(5,I+J)+A(2,2)                                          DGN13430
      B(6,I+J)=B(6,I+J)+A(3,2)                                          DGN13440
      B(7,I+J)=B(7,I+J)+A(1,3)                                          DGN13450
      B(8,I+J)=B(8,I+J)+A(2,3)                                          DGN13460
      B(9,I+J)=B(9,I+J)+A(3,3)                                          DGN13470
      RETURN                                                            DGN13480
      END                                                               DGN13490
      SUBROUTINE DISTR1(A,B)                                            DGN13500
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN13510
      DIMENSION A(3,3),B(9)                                             DGN13520
      A(1,1)=B(1)                                                       DGN13530
      A(2,1)=B(2)                                                       DGN13540
      A(3,1)=B(3)                                                       DGN13550
      A(1,2)=B(4)                                                       DGN13560
      A(2,2)=B(5)                                                       DGN13570
      A(3,2)=B(6)                                                       DGN13580
      A(1,3)=B(7)                                                       DGN13590
      A(2,3)=B(8)                                                       DGN13600
      A(3,3)=B(9)                                                       DGN13610
      RETURN                                                            DGN13620
      END                                                               DGN13630
      SUBROUTINE DISTR2(A,B)                                            DGN13640
      IMPLICIT REAL*8(A-H,O-Z)                                          DGN13650
      DIMENSION A(3,3),B(9)                                             DGN13660
      B(1)=A(1,1)                                                       DGN13670
      B(2)=A(2,1)                                                       DGN13680
      B(3)=A(3,1)                                                       DGN13690
      B(4)=A(1,2)                                                       DGN13700
      B(5)=A(2,2)                                                       DGN13710
      B(6)=A(3,2)                                                       DGN13720
      B(7)=A(1,3)                                                       DGN13730
      B(8)=A(2,3)                                                       DGN13740
      B(9)=A(3,3)                                                       DGN13750
```

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```
      RETURN
      END
      SUBROUTINE STRAIN(NOE,NDF,I,NPE)
C..........................................................................
C     THIS SUBROUTINE CALCULATES THE STRAIN AT GAUSSIAN POINT.
C..........................................................................
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XV(3,3)
      COMMON/STIF1/DSTIF(45,45),ELP(45),W(45),ELQ(45),WO(45),WI(45)
     1             ,W2(45),VAC(50,3),VEL(50,3),VACO(50,3),VELO(50,3)
      COMMON/MESH1/COOD(50,3),ELXYZ(9,3),NOD(9,9)
      COMMON/SHP/SF(9),GJINV(3,3),DSF(2,9)
      COMMON/ECV/V1(50,3),V2(50,3),V3(50,3),VA(50,3),VB(50,3),VC(50,3)
     1          ,V1A(50,3),V2B(50,3),V3C(50,3)
      DIMENSION GAUSS(4,4),DJ1(9,45),DJ2(9,45),DH1(9,45),DH(3,3),DC1(3)
     1          ,DH2(9,45),A1(6,6),A2(6,6),A3(6,6),SD(6,9),SD1(6,45),SD2
     2        (6,45),DC2(3),DB1(3),DB2(3),WT(4,4),ESTIF(9,45),SD3(6,45)
     3        ,TD(6,9),H(45,45),DK1(3,45),DK2(3,45),CSTIF(45,45),HT(3),
     4        HV(3)
      COMMON/STR/GU(9),GV(9),S1(6),S2(6),S3(6),XU(3,3),XV(3,3),GT(9),
                GS(9),GY(3,3)
      CALL ZERO(XV)
C
C     GENERATE THE LOCAL COORDINATE VECTORS.
C
      DO 351 I 1,NPE
      ND NOD(NOE,I)
      DO 451 K 1,3
      XV(K,1) XV(K,1)+SF(I)*VA(ND,K)
      XV(K,2) XV(K,2)+SF(I)*VB(ND,K)
      XV(K,3) XV(K,3)+SF(I)*VC(ND,K)
  451 CONTINUE
  351 CONTINUE
      SUM1 0.0
      SUM2 0.0
      SUM3 0.0
      DO 452 I 1,3
      SUM1 SUM1+XV(I,1)**2
      SUM2 SUM2+XV(I,2)**2
      SUM3 SUM3+XV(I,3)**2
  452 CONTINUE
      SUM1 DSQRT(SUM1)
      SUM2 DSQRT(SUM2)
      SUM3 DSQRT(SUM3)
      DO 453 I 1,3
      XV(I,1) XV(I,1)/SUM1
      XV(I,2) XV(I,2)/SUM2
      XV(I,3) XV(I,3)/SUM3
 453  CONTINUE
      DO 454 I 1,3
      DO 454 J 1,3
  454 XU(I,J) XV(J,I)
      DO 210 I 1,9
      GU(I) 0.0
      GV(I) 0.0
```

```
      GI(I) 0.0                                                     DGN14310
      GS(I) 0.0                                                     DGN14320
  210 CNTINUE                                                       DGN14330
   52 DO 93 I 1,NPT                                                 DGN14340
      ND NOD(NOE,I)                                                 DGN14350
      II NDF*(I-1)+1                                                DGN14360
      XA V3C(ND,1)-VC(ND,1)                                         DGN14370
      XB V3C(ND,2)-VC(ND,2)                                         DGN14380
      XC V3C(ND,3)-VC(ND,3)                                         DGN14390
      GV(1) GV(1)+DSI(1,I)*W(II)                                    DGN14400
      GV(2) GV(2)+DSI(2,I)*W(II)                                    DGN14410
      GV(4) GV(4)+DSI(1,I)*W(II+1)                                  DGN14420
      GV(5) GV(5)+DSI(2,I)*W(II+1)                                  DGN14430
      GV(7) GV(7)+DSI(1,I)*W(II+2)                                  DGN14440
      GV(8) GV(8)+DSI(2,I)*W(II+2)                                  DGN14450
      GV(3) GV(3)+SI(I)*XA*0.5*T                                    DGN14460
      GV(6) GV(6)+SI(I)*XB*0.5*T                                    DGN14470
      GV(9) GV(9)+SI(I)*XC*0.5*T                                    DGN14480
      GI(1) GI(1)+DSI(1,I)*XA*0.5*T                                 DGN14490
      GI(2) GI(2)+DSI(2,I)*XA*0.5*T                                 DGN14500
      GI(4) GI(4)+DSI(1,I)*XB*0.5*T                                 DGN14510
      GI(5) GI(5)+DSI(2,I)*XB*0.5*T                                 DGN14520
      GI(7) GI(7)+DSI(1,I)*XC*0.5*T                                 DGN14530
      GI(8) GI(8)+DSI(2,I)*XC*0.5*T                                 DGN14540
   93 CONTINUE                                                      DGN14550
   73 CALL DISTR1(GY,GV)                                            DGN14560
      CALL MXMUT2(GJINV,XV,XU,GY)                                   DGN14570
      CALL DISTR2(GY,GU)                                            DGN14580
      CALL DISTR1(GY,GI)                                            DGN14590
      CALL MXMUT2(GJINV,XV,XU,GY)                                   DGN14600
      CALL DISTR2(GY,GS)                                            DGN14610
C...................................................................DGN14620
C     STRAIN   S1 + Z * S2                                          DGN14630
C...................................................................DGN14640
   96 S1(1) GU(1)                                                   DGN14650
      S1(2) GU(5)                                                   DGN14660
      S1(3) GU(2)+GU(4)                                             DGN14670
      S1(4) GU(3)+GU(7)                                             DGN14680
      S1(5) GU(6)+GU(8)                                             DGN14690
      S2(1) GS(1)                                                   DGN14700
      S2(2) GS(5)                                                   DGN14710
      S2(3) GS(2)+GS(4)                                             DGN14720
      S2(4) GS(3)+GS(7)                                             DGN14730
      S2(5) GS(6)+GS(8)                                             DGN14740
      IF(NIMAX.EQ.1) GO TO 721                                      DGN14750
      S1(1) S1(1)+0.5*(GU(1)**2+GU(4)**2+GU(7)**2)                  DGN14760
      S1(2) S1(2)+0.5*(GU(2)**2+GU(5)**2+GU(8)**2)                  DGN14770
      S1(3) S1(3)+GU(1)*GU(2)+GU(4)*GU(5)+GU(7)*GU(8)               DGN14780
      S1(4) S1(4)+GU(1)*GU(3)+GU(4)*GU(6)+GU(7)*GU(9)               DGN14790
      S1(5) S1(5)+GU(2)*GU(3)+GU(5)*GU(6)+GU(8)*GU(9)               DGN14800
      S2(1) S2(1)+GU(1)*GS(1)+GU(4)*GS(4)+GU(7)*GS(7)               DGN14810
      S2(2) S2(2)+GU(2)*GS(2)+GU(5)*GS(5)+GU(8)*GS(8)               DGN14820
      S2(3) S2(3)+GU(1)*GS(2)+GU(2)*GS(1)+GU(4)*GS(5)+GU(5)*        DGN14830
     1     GS(4)+GU(7)*GS(8)+GU(8)*GS(7)                            DGN14840
      S2(4) S2(4)+GU(1)*GS(3)+GU(3)*GS(1)+GU(4)*GS(6)+GU(6)*        DGN14850
```

```
     1      GS(4)+GU(7)*GS(9)+GU(9)*GS(7)                            DGN14860
      S2(5)=S2(5)+GS(8)+GU(3)*GS(2)+GU(2)*GS(3)+GU(6)*GS(5)+GU(5)*   DGN14870
     1      GS(6)+GU(9)*GS(8)+GU(8)*GS(9)                            DGN14880
      S3(1)=0.5*(GS(1)**2+GS(4)**2+GS(7)**2)                         DGN14890
      S3(2)=0.5*(GS(2)**2+GS(5)**2+GS(8)**2)                         DGN14900
      S3(3)=GS(1)*GS(2)+GS(4)*GS(5)+GS(7)*GS(8)                      DGN14910
      S3(4)=GS(1)*GS(3)+GS(4)*GS(6)+GS(7)*GS(9)                      DGN14920
      S3(5)=GS(2)*GS(3)+GS(5)*GS(6)+GS(8)*GS(9)                      DGN14930
      RETURN                                                         DGN14940
      END                                                            DGN14950
```

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